ECONOMIC RESEARCH
FEDERAL RESERVE BANK OF ST. LOUIS

WORKING PAPER SERIES

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Authors	Alex Cukierman, and G.J. Santoni
Working Paper Number	1987-002A
Creation Date	January 1987
Citable Link	https://doi.org/10.20955/wp.1987.002
Suggested Citation	Cukierman, A., Santoni, G., 1987; Some Problems with Current Price Indexes, Federal Reserve Bank of St. Louis Working Paper 1987-002. URL https://doi.org/10.20955/wp.1987.002

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SOME PROBLEMS WITH CURRENT PRICE INDEXES

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87-002

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0858K/0207R
GJS/plm
4/22/87
Draft 3

Some Problems With Current Price Indexes

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1. INTRODUCTION

Price indexes are widely used economic tools. They are regularly employed to draw inferences regarding welfare across both time and location from observations of nominal income.[1/] Economic agents use them to distinguish aggregate from relative price changes. In part, they determine the choice set that agents perceive and so importantly influence decisions to consume, to save, and to invest.[2/]

Various price indexes differ in terms of their construction and it is generally reorganized that selection of the appropriate index largely depends on the particular purpose for which it is to be used.[3/] This note contends that currently published price indexes are not well suited to the analysis of an important class of problems.

We show that the application of these price indexes can bias the analysis if individuals can substitute freely between pure consumption goods and goods with lives extending beyond the current period.[4/]

The problem with commonly used price indexes is that they are incomplete. The prices of current service flows receive very heavy weights while relatively low weights are assigned to the prices of more durable goods and assets (the present prices of future consumption flows). This problem has not gone unnoticed in the previous literature. Contrary to present practice, this literature suggests that measurement of the price level should be based on "wealth-like magnitudes not income magnitudes."[5/] Our purpose is to formally compare the price level of economic theory to price indexes of the type currently published. The analysis indicates that current service flow price indexes produce biased estimates of both the general level of prices and the rate of inflation.[6/] These biases are shown to depend on the ex ante real rate of interest and rate of gross investment.

2.0 THE ANALYSIS

Call the price level of economic theory the Fisherian price level. It measures the money cost of a given level of utility.[7/] Utility depends on both the flow of services immediately consumed and on the expected flow of future consumption services.[8/] The following analysis assumes that markets exist for every consumption service flow to be delivered at every moment in time, that individuals are free to transact in any of these markets, that the individual's time horizon is infinite, that the yield curve is flat and that gross investment is positive but net investment and saving are zero. Consumption is equal to permanent income.

The Fisherian price level in the base period, P_B^T, is given by equation 1. The per period quantity of permanent consumption services expected in the based period is C. The current and expected future money price of these services is P per unit and the *ex ante* real interest rate is r $(0 < r < \infty)$.[9/]

$$P_B^T = PC + \frac{PC}{(1+r)} + \frac{PC}{(1+r)^2} + \cdot\cdot\cdot\cdot = PC\left(1 + \frac{1}{r}\right) \qquad (1)$$

The capital stock (K) is the present value of the permanent consumption stream $(K = C/r)$ so equation 1 may be written as [10/]

$$P_B^T = PC + PK. \qquad (2)$$

The Fisherian price level depends on the present money cost of the stream of permanent consumption. In contrast, currently published price indexes include the present money cost of goods immediately consumed and the present money cost of the capital stock _that is produced and sold in the current period_.[11/] Roughly, these price indexes take account of the cost of current consumption and current period gross investment. In the world we live in, this represents only a portion of the present cost of the stream of permanent consumption. As a result, currently published price indexes are "incomplete."[12/]

2.1 The Bias In the Measured Price Level

If the rate of real gross investment is given by δ, equation 3 approximates the current technology employed to measure the price level.

$$P_B^M = PC + \delta \; PK = PC\,(1 + \delta/r) \qquad (3)$$

$$0 < \delta < 1$$

The measured price level (P_B^M) is a downward biased estimate of the Fisherian price level (P_B^T) given the range of values that r and δ are restricted to. This bias (B_o) is shown in equation (4).

$$B_o = \frac{P_B^M}{P_B^T} = \frac{PC\,(1 + \delta/r)}{PC\,(1 + 1/r)} = \frac{r + \delta}{r + 1} < 1 \qquad (4)$$

The downward bias in the measured price level is smaller the larger are δ and r. A larger δ means that a larger portion of the cost of the existing capital stock (PK) is included in the measured price level so the measured price level more closely approximates the Fisherian price level. A larger r means that the present cost of the stream of permanent consumption (the capital stock) is lower. As a result, both the measured and theoretical price levels are lower but P_B^M is higher relative to P_B^T because a higher r raises the numerator in (4) relative to the denominator.

2.2 Changes In the Money Supply

Suppose a change in the money supply changes the money price of consumption services (P) while C, r and δ are constant. The elasticity of both the Fisherian and measured price levels with respect to the price of consumption services is

$$\frac{\partial P^T}{\partial P} \frac{P}{P^T} = \frac{P(C+K)}{P(C+K)} = 1 = \frac{\partial P^M}{\partial P} \frac{P}{P^M} = \frac{P(C+\delta K)}{P(C+\delta K)} . \qquad (5)$$

Both measured and Fisherian price levels increase in proportion to the increase in the price of present consumption goods. In this case, the percentage change in P^M accurately measures the percentage change in P^T.[13/] On these grounds, inflations (deflations) that are purely monetary in nature are accurately measured by current service flow price indexes.

Alchian and Klein (1973) appear to draw an over strong conclusion when they say that "They, therefore, often are used as measures of inflation and often are targets or indicators of monetary and fiscal policy. Nevertheless, these price indices, which represent measures of current consumption service prices and current output prices, are theoretically inappropriate for the purpose to which they are generally put (p. 173)." Rather, they are inappropriate measures of inflation given certain events as detailed in the following analysis.

2.3 <u>The Effect of Changes In r on the Price Level</u>

As mentioned above, a change in the interest rate changes both the Fisherian price level and the measured price level. However, the change in the measured price level is not proportional to the change in the Fisherian price level. To see this, differentiate equations (1) and (3) and convert to elasticitics.

$$\eta_r^T = \frac{\partial P^T}{\partial r}\frac{r}{P^T} = -\frac{PC}{r^2}\frac{r}{PC(1+1/r)} = -\frac{1}{1+r} \qquad (6)$$

$$\eta_r^M = \frac{\partial P^M}{\partial r}\frac{r}{P^M} = -\frac{\delta PC}{r^2}\frac{r}{PC(1+\delta/r)} = -\frac{\delta}{\delta+r} \qquad (7)$$

It can be shown that $\eta_r^M > \eta_r^T$ if, as assumed above, $\delta < 1$.[14/] Potentially, this is of some importance. For example, an increase in r causes the measured price level to rise <u>relative to</u> the Fisherian price level. The measured price level is viewed by both private individuals and policy makers as an important piece of economic information. Among other things, changes in it help distinguish aggregate from relative price changes.

The bias introduced by changes in r makes these current service flow price indices less reliable guides in making this distinction.

While this bias is potentially important, it may not be of much practical importance. Equation 8 expresses the bias (B_1) as the difference between the interest elasticities of the measured and Fisherian price levels. Table 1 presets some ranges for these elasticities and for B_1 assuming that r ranges from 1.0 percent to 10.0 percent while δ ranges from 5.0 percent to 30.0 percent.

$$B_1 = \eta_r^M - \eta_r^T = -\frac{\delta}{\delta+r} + \frac{1}{1+r} = \frac{r(1-\delta)}{(1+r)(r+\delta)} > 0 \qquad (8)$$

The table 1 calculations indicate that the interest elasticity of the measured price level ranges between -.33 and -.97. For example, a one percent increase in r reduces the measured price level by no more than .97 percent and no less than .33 percent depending on the initial value of r and value of δ. On the other hand, the range for the interest elasticity of the Fisherian price level is much narrower. For example, a one percent increase in r reduces the Fisherian price level by no more than .99 percent and no less than .91 percent. The bias (B_1) ranges from .02 to .58 suggesting that, as a practical matter, a change in r may result in a serious measurement error.

Given the table 1 calculations, it appears that B_1 increases with r and decreases with δ and it can be shown that this is normally the case.15/

2.4 The Rate of Inflation and Changes In r.

As shown above (section 2.2), inflations (deflations) that are purely monetary in nature are accurately measured by the rate of change in the measured price level. However, should r change during an inflationary (deflationary) episode, both the measured and Fisherian price levels record a once-and-for-all change with the measured price level changing relative to the Fisherian price level.

When r changes, the rate of change recorded by the Fisherian price level deviates temporarily from the change produced by the purely monetary factors. for example, if purely monetary factors are producing a 5 percent rate of change in the Fisherian price level, other things the same, an increase in r will reduce the rate of change in the Fisherian price level during the time period when the increase in r occurs. Once r has reached its new level, the rate of change returns to 5 percent. Equation (9) makes use of equations (5) and (6) to express this result.

$$\frac{\Delta P^T}{P^T} = \frac{\Delta P}{P} + \eta_r^T \frac{\Delta r}{r} \qquad (9)$$

Given the range of values for η_r^T presented in table 1 and an assumed monetary inflation rate of 5 percent, a one percent increase in r causes a temporary fall in the rate of change in the Fisherian price level to about 4.1 percent.

A similar qualitative result holds for the rate of change recorded by the measured price level. An increase in r causes the measured rate of inflation to fall below the rate of inflation produced by purely monetary factors. This is shown in equation (10).

$$\frac{\Delta P^M}{P^M} = \frac{\Delta P}{P} + \eta_r^M \frac{\Delta r}{r} \qquad (10)$$

Furthermore, since $\eta_r^T < \eta_r^M$ the percentage change in the measured price level will exceed the percentage change in the theoretical price level when r changes. When r rises, the rate of inflation in the measured price level overstates the rate of inflation in the Fisherian price level and conversely.16/ Equation (11) expresses this bias (B_2) as the difference between equations (9) and (10).

$$B_2 = \frac{\Delta P^M}{P^M} - \frac{\Delta P^T}{P^T} = (\eta_r^M - \eta_r^T) \frac{\Delta r}{r} = B_1 \frac{\Delta r}{r} \qquad (11)$$

Since B_1 is always positive, B_2 is positively related to the change in r. The computed values in table 1 can be used to give some ranges for the size of B_2. For example, a 1.0 percent increase in r results in measured inflation overstating the Fisherian rate by no more than .58 percentage points and no less than .02 percentage points.

2.5 The Effect of Changes in δ

In contrast to r, which entered the expression for both the Fisherian and measured price level, the rate of gross investment (δ) only appears in the expression for P^M. Consequently, when δ changes, the Fisherian price level is constant but the measured price level changes. Proceeding as before, the bias (B_3) introduced by changes in δ is given below.

$$\eta_\delta^T = \frac{\partial P^T}{\partial \delta} \frac{\delta}{P^T} = 0$$

$$\eta_\delta^M = \frac{\partial P^M}{\partial \delta} \frac{\delta}{P^M} = \frac{\delta}{r+\delta} > 0$$

$$B_3 = \eta_\delta^M - \eta_\delta^T = \frac{\delta}{r+\delta} > 0 \tag{12}$$

Notice that $\eta_\delta^M = B_3 = -\eta_r^M$ so, by reversing the sign, the computations for η_r^M in table 1 give ranges for B_3.

As with r, should δ change during an inflationary (deflationary) episode, the measured rate of inflation deviates from the rate of inflation in the Fisherian price level. This bias (B_4) is shown in equation (13).

$$B_4 = \frac{\Delta P^M}{P^M} - \frac{\Delta P^T}{P^r} = (\eta_\delta^M - \eta_\delta^T) \frac{\Delta \delta}{\delta} = B_3 \frac{\Delta \delta}{\delta} \qquad (13)$$

Since $B_3 > 0$, B_4 is positively related to the change in δ. An increase in δ, for example, causes measured inflation to overstate the Fisherian rate. A 1.0 percent increase in δ results in an overstatement that is no greater than .97 percentage points and no less than .33 percentage points, given the assumed ranges for r and δ.

3.0 SOME APPLICATIONS

The bias in the measured price level is relevant for a number of economic problems. As mentioned above, it can confuse the distinction between aggregate and relative price changes. More importantly, perhaps, it may influence the speed at which permanent and temporary changes are distinguished. Whether economic agents are able to make this distinction quickly or slowly is clearly important for the time path of real output and employment. The literature on this issue has argued that "the permanent and transitory components of the shocks that affect the economy are not known in advance and are not revealed for some time after they occur. People must solve an inference problem to distinguish permanent and transitory components of the data they observe."[17] Changes in r or δ shock both the Fisherian price level and temporarily shock its rate of change. If individuals directly observed this price level, they are still confronted with the problem of distinguishing the permanent and temporary components of the shock. As this note points out, the measured price levels that people observe differ from the price level of economic theory. We speculate that this further complicates the problem of distinguishing between permanent and transitory components of the shock.

The above would not be practically important if direct observations of the *ex ante* real interest rate or market value of the capital stock were possible. However, both are emirically illusive variables. Economic agents come to a realization that these variables have changed over time and the eventual perception of the change is always clouded.

Finally, the above analysis of the bias contained in current service flow price indexes may help explain why there is a general reluctance to tie nominal values specified in long term contracts to these price indexes through the use of escalator clauses. While some countries have experimented with these clauses, they have not come into wide use. This is true even in countries like the United States that have experienced considerable variation in their *measured* rates of inflation in the past twenty or so years. It may be that economic agents recognize that an important part of the change in the price level that is measured between any two points in time is likely to be temporary and due to "special factors" of the type discussed in this note.

FOOTNOTES

1/ See Fisher (1963) who identifies six broad purposes for which price indexes may be used: "to compare the prices, in different places, of the goods exchanged; of the capital goods; and of the income goods; and the same three groups of goods at different times (p. 205)."

2/ See Brunner, Cukierman and Meltzer (1980) for a discussion of the importance of perceptions regarding the permanence of a change for economy wide resource allocation decisions.

3/ See Ruist (1968), Konüs (1939), Fisher (1927) and (1963), pp. 204-5.

4/ Fisher (1963) considers the application of prices indexes to this type of problem as "perhaps the most important purpose" to which index numbers may be put.

5/ A most important recent contribution to the analysis of this problem is by Alchian and Klein (1973) who argue that "these price indices, which represent measures of current consumption service prices and current output prices, are theoretically inappropriate for the purpose to which they are generally put (p. 173)." A similar conclusion is expressed by Pigou (1960), p. 36, Fisher (1963), pp. 213-14 and Samuelson (1961), pp. 32-57.

Samuelson concludes that "when we work with simple exact models, in which no extraneous statistical difficulties of measurement could arise, we find that the only valid approximation to a measure of welfare comes from computing wealth-like magnitudes not income magnitudes (emphasis in original, p. 57)."

6/ Alchian and Klein (1973), pp. 178-81 discuss these biases. Our analysis draws them out somewhat more sharply and provides some crude estimates of their magnitude.

7/ See Alchian and Klein (1973), p. 174; Konüs (1939), p. 10 and Ruist (1968), p. 155.

8/ The price level of economic theory measures the money cost of permanent consumption. See Friedman (1957).

9/ The analysis ignores the expected rate of inflation, π, because including it does not change the problem. If $\pi \neq 0$, the nominal rate is $i = r + \pi$ and P_T is

$$P_T = PC + \frac{(1+\pi)PC}{1+i} + \frac{(1+\pi)^2 PC}{(1+i)^2} + \ldots =$$

$$= PC + \frac{(1+\pi)PC}{(1+\pi)(1+r)} + \frac{(1+\pi)^2 PC}{(1+\pi)^2(1+r)^2} + \ldots$$

$$P_T = PC + \frac{PC}{(1+r)} + \frac{PC}{(1+r)^2} + \ldots$$

which is the same as equation 1.

10/ The analysis assumes that consumption occurs at the beginning of each period. Thus, current period consumption is not included in the value of the capital stock.

11/ The GNP deflator is an example. This price index measures the cost of all goods and services produced and sold during the relevant quarter. Most of these are consumption goods. However, some are capital goods.

12/ See Alchian and Klein (1973), p. 173.

13/ Alchian and Klein (1973) did not focus on this case. Rather, they assumed that "monetary impulses are transmitted to the real sector of the economy by producing transient changes in the relative prices of service flows and assets (i.e. by producing transient changes in 'the' real rate of interest), ... (p. 173)." However, the evidence regarding the effect of monetary impulses on the real rate of interest is, at best, spotty. See Brown and Santoni (1983), pp. 16-25 and Melvin (1983), pp. 188-202.

14/ $\eta_r^M \gtrless \eta_r^T$ as $-\delta/(\delta+r) \gtrless -1/(1+r) \iff$

$\delta(1+r) \lessgtr \delta+r \iff \delta \lessgtr 1.$

Since $\delta < 1$ by assumption, $\eta_r^M > \eta_r^T$.

15/ $B_1 = \dfrac{r(1-\delta)}{(1+r)(r+\delta)}$

$$\frac{\partial B_1}{\partial \delta} = - \frac{r}{(1+r)} \frac{[(r+\delta)+r(1-\delta)]}{(r+\delta)^2} < 0$$

$$\frac{\partial B_1}{\partial r} = \frac{(1-\delta)(\delta - r^2)}{[(1+r)(r+\delta)]^2}$$

For reasonable values of r and δ, $\partial B_1/\partial r > 0$.

16/ Notice that the rate of change in the measured price level more closely approximates the rate of inflation induced by purely monetary forces in these circumstances than does the rate of change in the theoretical price level. This is because the monetary rate of inflation, like the measured rate of inflation, is a biased estimate of the change in the nominal cost of maintaining a given level of utility when r changes.

17/ Brunner, Cukierman and Meltzer (1980), p. 492.

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Table 1
Some Ranges for the Interest Elasticities of the Measured and Theoretical Price Levels

δ	r	η_r^M	η_r^T	$B_1 = \eta_r^M - \eta_r^T$
0.05	0.01	−0.83	−0.99	0.16
0.05	0.10	−0.33	−0.91	0.58
0.30	0.01	−0.97	−0.99	0.02
0.30	0.10	−0.75	−0.91	0.16