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A MODEL OF GLOBAL AGGREGATE SUPPLY AND DEMAND USING VECTOR AUTOREGRESSIVE TECHNIQUES

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I. Introduction

Major oil-importing economies' apparent stagflationary responses to the oil price shocks of the 1970s led economists to emphasize the roles of both aggregate supply and aggregate demand in macro models. These models have generally focused on a single national economy or a small group of separately modeled national economies.[1/] Global impacts can be inferred only under restrictive assumptions by adding up the outcomes of these national models.

Two traditions exist to explicitly model the global economy. One follows the path pioneered by the LINK project with detailed macroeconometric models of many national economies linked by a trade matrix and set of exchange rate equations.[2/] The foreign exchange rate equations of the LINK-type models, however, have poor forecasting records. For example, they generally failed to predict the rise of the U.S. dollar in the 1983-85 period.[3/] The LINK-type models also have implications for the global economy only as an after-the-fact summation of the results of the national level models.

Sim's (1980) critique of the "incredible identifying restrictions" associated with large-scale, macro models is relevant to LINK-type models. He argued that serious problems exist due to excessive use of a priori restrictions, for example on lag lengths and on excluded variables, and proposed using simpler models in a vector autoregressive framework. While Hamilton (1983), Lacker (1984) and Burbidge and Harrison (1984) have used this approach in their national level studies of the impacts of oil price changes, the vector autoregressive approach has generally not been used to model the global economy.

The major alternative to the LINK-type approach for modeling the world economy focuses directly on global aggregates.[4/] This approach is the global equivalent of modeling a national economy directly rather than separately modeling individual sectors and then summing the results.

The first modeling efforts at the global level have shown a monetarist orientation and have found strong relations between global money and a global price index.[5/] These studies generally do not consider supply-side effects. The focus instead is on global inflation as a determinant of national inflation rates under a fixed exchange rate system. Under flexible exchange rates, the transmission mechanism no longer requires a world inflation to domestic inflation linkage.

We present a straightforward theoretical model of global aggregate supply and demand and estimate a four variable VAR based on the theoretical model.[6/] Impulse response functions and variance decompositions are presented as alternate representations of the empirical relationships. Our estimation results are, in general, consistent with the theoretical model.[7/]

II. Theory

The theoretical model is based on global aggregate supply and demand and focuses on four variables: real gross global product, a global consumer price index, global money supply and a global energy price index. The latter two are assumed to directly influence global aggregate demand and supply. The actual output and price level pairs are assumed to represent short-run equilibria of the aggregate supply and demand curves.

Following Fischer (1985), global aggregate demand is assumed to equal nominal world product. This implies a downward-sloping, rectangular hyperbola in price-real product space with the role of the global money supply indicated by the quantity equation.

(1) $AD = MV = PY,$

where AD is global aggregate demand, M is the global money supply, V is the velocity of money, P is the price level and Y is real output. Rewriting in natural logarithmic form (lower case letters indicate natural logs):

(2) $p = m + v - y.$

Changes in M are assumed not fully offset by changes in V. Thus, money directly influences aggregate demand. Furthermore, nonmonetary disturbances to aggregate demand must operate through changes in velocity.

Following Bruno and Sachs (1982), we assume that global aggregate supply is derived from short-run, cost-minimization solutions based upon a three-factor, weakly separable, two-level, aggregate CES production function, subject to Harod-neutral technical change. Thus:

(3) $Y = Y\,[A(K, L), E]$,

where E, the energy input, is weakly separable from real value-added A with δ the elasticity of substitution between A and E and σ, that between K and L. Y is real output and K and L are capital and labor, respectively. Furthermore, let us associate α, β and γ with the output elasticities of labor, energy and capital, respectively. Furthermore, let W be the nominal wage rate, Π_{ε} equal the real price of energy and λ be the rate of labor-augmenting technical change. Then, following Bruno and Sachs, we obtain the following locally linearized output supply equation, assuming global profit-maximizing behavior on the supply side:

(4) $\ln AS = y = \psi - (\alpha\sigma/\gamma)\ (w - p - \lambda t) - (\beta\ \eta\sigma/\gamma)\ \Pi_E + k$,

where $\eta = (1 - \beta)^{-1}[\alpha + \psi^{-1}\ \sigma\gamma]$ and $0 < n \leq 1$ and $\sigma \geq 0$ and where ψ is an inessential constant. w and k are again in logs, and t is a time trend. Equation (4) can be rewritten as:

(5) $p = w - \lambda\ t - (\gamma/\alpha\sigma)\ (\psi + k - y) + (\beta\ \eta/\alpha)\ \Pi_E$,

Clearly, in this form we have an upward-sloping AS curve. As is well known, this result depends upon some degree of stickiness of nominal wages, at least in the short run. Such stickiness could arise from institutional rigidities such as long-term nominal contracts.

Constant real wages are also possible, presumably reflecting an assumption of constantly fulfilled rational expectations with no institutional labor market rigidities. This will lead to a vertical AS curve with no determinate equivalent to equation (5). If w_p equals the constant real wage, then:

$$(6) \quad \ln AS = y = \psi - (\alpha\sigma/\gamma)\ (w_p - \lambda t) - (\beta\eta\ \sigma/\gamma)\ \pi_E + k.$$

Note that, irrespective of whether or not the AS curve is vertical, an increase in π_E, the energy price index, will lead to an adverse, backward shift or the AS curve.

Let us now consider the reduced form solutions under the two alternative AS forms. Beginning with the case of sticky nominal wages, we simultaneously solve equations (2) and (4) to obtain the following price-output equilibrium pair:

$$(7) \quad p^* = [(\alpha\sigma/\gamma)\ (w - \lambda t) - \psi + (\beta\eta\sigma/\gamma)\pi_E - k + m + v][1 + (\alpha\sigma/\gamma)]^{-1}$$

$$(8) \quad y^* = [\psi - (\alpha\sigma/\gamma)\ (w - m - v - \lambda t) + k - (\beta\eta\sigma/\gamma)\ \pi_E]\ [1 + (\alpha\sigma/\gamma)]^{-1}.$$

Differentiating these leads to the following results:

$$(9) \quad \frac{\partial p^*}{\partial m} = [1 + (\alpha\sigma/\gamma)]^{-1} > 0$$

$$(10) \quad \frac{\partial p^*}{\partial \Pi_E} = (\beta\eta\sigma)/(\gamma + \alpha\sigma) > 0$$

(11) $$\frac{\partial y^*}{\partial m} = \alpha/[\alpha + \gamma/\sigma] > 0$$

(12) $$\frac{\partial y^*}{\partial \Pi_E} = -(\beta\eta)/[\alpha + \gamma/\sigma] < 0 .$$

Adverse supply shocks due to energy price increases will tend to raise prices and lower output in equilibrium. In contrast, prices and output will tend to change in the same direction as shifts of aggregate demand due to money supply changes.

In the case of real wage constancy and hence of neutral money we have the following reduced form equations:

(13) $$p^* = m + v - \psi + (\alpha\sigma/\gamma)(w_p - \lambda t) + (\beta\eta\sigma/\gamma)\Pi_E + k$$

(14) $$y^* = \psi - (\alpha\sigma/\gamma)(w_p - \lambda t) - (\beta\eta\sigma/\gamma)\Pi_E + k$$

Differentiating these we obtain:

(15) $$\frac{\partial p^*}{\partial m} = 1$$

(16) $$\frac{\partial p^*}{\partial \Pi_E} = (\beta\eta\sigma) / \gamma > 0$$

(17) $$\frac{\partial y^*}{\partial m} = 0$$

(18) $$\frac{\partial y^*}{\partial \Pi_E} = -(\beta\eta\sigma) / \gamma < 0 .$$

With a vertical AS curve, changes in the money supply do not affect output but are fully reflected in price level changes. The impact of a supply change due to an energy price change will have similar effects as in the previous case with respect to signs, although the magnitudes will differ (unless $\alpha = 0$).

The discussion to this point considers the short-run static adjustments of price and output to presumably exogenous changes in money supply and energy prices without considering the possible endogeneity of the latter two.[8/] To consider the longer-run behavior of the model, we need to consider the feedbacks from price and output changes onto the money supply and energy prices. We can make no unambiguous predictions about the precise nature of those feedbacks, however.

With respect to the global money supply, there are three possible reactions of monetary policymakers. The first assumes monetary policymakers do not react to any macro aggregate. Money is a random walk or follows an autoregressive process. In this case, the money stock could be modeled simply with an ARIMA. Second, the monetary authorities may react in an *ex post* countercyclical fashion. Changes in the money supply respond negatively to lagged changes in price and output or perhaps are used to offset energy price changes. And third, monetary policy may be accommodative, responding positively to lagged changes in output and prices. In the latter two cases, some form of reaction function is required.

Alternate views also exist on the determinants of energy prices. In a long-run equilibrium growth model, real energy prices would increase smoothly at the true rate of discount according to the Hotelling Rule (1932). This would suggest an autoregressive component to energy prices. In a more complex world of sequential short-run equilibria as proposed here, the demand for energy and hence its real price should fluctuate with real output.[9/]

Beenstock and Dicks (1983) and Bruno and Sachs (1985) present a sharply contrasting view of energy price determination. Energy prices are assumed to be totally exogenous, subject exclusively to the whims of OPEC or other market manipulators or to purely random factors. To the extent that the energy market is controlled by OPEC, the determinants of OPEC's reaction functions may be as uncertain as those of the money supply. In particular, with respect to global inflation, we might expect almost any response by real energy prices.[10/]

Finally, we have some uncertainty about the dynamic impacts of price and output on each other. Examination of the above reduced form equations suggests a possible direct link from prices to output in the case of sticky prices as inflation lowers real wages. To the extent that short-run rigidities exist and to the extent that the system is subject to random shocks, prices and output may have distributed lag effects on each other. Changes in prices and output may also affect each other if those changes alter expectations about future global macroeconomic policies. Thus, accelerating inflation may induce expectations of future contractionary policies. These anticipations may, in turn, reduce aggregate demand by lowering velocity, for example.

Before presenting the empirical results, a word is in order on aggregation. Is it meaningful to consider GNP and inflation at the global level or do ineradicable index number problems make these variables undefinable under floating exchange rates as Harry Johnson (1977) has argued? Under fixed exchange rates, the concepts of global inflation and output have been used extensively. For example, Genberg and Swoboda (1977) and Gray, Ward and Zis (1977) have argued, based on the monetary approach to the balance of payments, that inflation rates will tend to converge among countries except for productivity change differences and non-traded goods price differences. Global inflation will drive national inflations. In a world of fixed exchange rates, the global macroeconomy is just a national macroeconomy with regional differences. The index number problems are not different from aggregation from the regional to the national level.

Under flexible exchange rates, the issue is considerably more complex. Flexible exchange rates in theory insulate individual national economies from the inflationary excesses of their trading partners. Inflation is not transmitted between countries, and the concept of global inflation would, arguably, have no meaning. In practice, however, national policymakers have not generally allowed freely fluctuating exchange rates. A careful reading of yearly issues of Exchange Arrangements and Exchange Restrictions: Annual Report, the IMF's summary of exchange rate practices, suggests that very few countries have, in fact, allowed freely floating rates. Most have either continued with fixed exchange rates or have a so-called managed float. One might argue that the aggregation problem would remain if only the largest countries were to float. Batten and Ott (1985) have demonstrated, however, that even a group of seven large erstwhile freely floating countries, in fact, have not fully availed themselves of the insulating properties of flexible exchange rates. Furthermore, Farber, Roll and Solnik (1977) and Sheehan (1986) both emphasize that neither completely fixed nor completely flexible exchange rates have existed. Thus, to the extent that exchange rate practices can be approximated by fixed exchange rates, the aggregation problem is relatively unimportant. In addition, the increasing "globalization" of markets for basic commodities makes it increasingly important to focus on global forces influencing supply and demand rather than national or local factors.[11/]

III. Data and Methodology

Our reduced form equation system is estimated using vector autoregressive analysis (VAR). Quarterly data is used for aggregate OECD industrial production (Q), the OECD CPI (P), OECD M1 (M) and an OECD energy price index (P_E), the OECD being taken as an approximation of the global economy. All data is obtained from OECD Main Economic Indicators, and the methodology for the construction of the aggregate series can be found in OECD Sources and Methods (1977). Our sample period runs from the second quarter of 1971 through the first quarter of 1985, the longest possible sample for which data exists on all series. All variables have been transformed into the first differences of natural logs.

A major issue in VAR estimation involves the determination of the appropriate lag structure. Kling and Bessler (1985) describe a number of alternate procedures for ascertaining the lag length. To insure that our results are not procedure-sensitive, we employ two techniques. To the extent that the results are consistent, we can be more confident in the conclusions.

The first approach uses an unrestricted VAR process with each variable appearing in all equations with the same number of lags, following Sims (1981) and Litterman (1985).[12/] The results using this approach are labeled SL. OLS is the appropriate estimation technique when using this approach since the right-hand side variables are identical in all equations. The disadvantages of this approach are well documented. There is a possibility for overparameterization of the model. In addition, too many lags of some variables and too few of others may be included in some equations.

The second approach follows Hsiao (1981) and Fackler (1985) and uses Akiake's Final Prediction Error (FPE) criterion to determine included variables and appropriate lag lengths. The results using this approach are labeled FPE. The FPE approach first estimates a sequence of autoregressions for each variable:

$$(19) \quad Y(i,t) = \alpha(0) + \sum_{r=1}^{R} \alpha(r)\, Y(i, t-r) + e(t) .$$

The maximum lag length, R, is sequentially increased from one to some *a priori* maximum (here generally eight quarters). The appropriate lag length minimizes the FPE, thus balancing the risk due to bias when a lower order is selected and the risk due to increased variance when a higher order is selected. Other variables are then sequentially added to equation (19), and the lag length for each additional variable is determined as above. Finally, the resulting system is simultaneously estimated and then tested by deliberately under- and overfitting.

The SL estimation includes three lags of all variables. The restriction that the fourth lags were jointly insignificant could not be rejected. In contrast, a χ^2 test of the significance of the third lags clearly rejects the null hypothesis of insignificance.[13/]

The significance levels for the individual SL results are presented in table 1. The results suggest an autoregressive component to the behavior of all series but relatively few (3 out of a possible 12) significant causal relations.[14/] Apparently money growth, inflation and output growth are all influenced by energy price changes. Real energy prices, in turn, are influenced solely by inflation. Clearly, the lack of significance of money growth in influencing inflation would be a source of major concern for a monetarist. This result is discussed in more detail below.

The FPE results suggest the presence of substantially more causal flows. The estimated FPE lag lengths are presented in the top half of table 2. A zero coefficient indicates the FPE with the variable included was no lower than with it excluded or that the overfitting and underfitting tests indicated it was insignificant at the 10 percent level. Thus, variables with zero coefficients were excluded from the final equations.

The significance levels for the FPE results are presented in the bottom half of table 2, included in the summary of some of the overfitting and underfitting tests. As with the SL results, changes in energy prices influence money growth, inflation and output growth and are influenced by inflation. In addition, money growth is also affected by output growth and inflation, while inflation also responds to money growth and output growth. These findings should serve as a warning to other researchers that the choice of lag length selection procedure may substantially alter subsequent causality conclusions.[15/] The results further suggest that the FPE procedure may be better at ferreting out relationships since extraneous lags are presumably excluded, thus increasing the efficiency of the results, while perhaps also being less likely to truncate the lag distribution at too low a level. The difference in the SL and FPE results certainly argue for caution in interpreting the conclusions.

The results in tables 1 and 2 indicate the statistical significance of the relationships and do not measure the economic significance of the results. Statistically important relationships may be trivial in magnitude, while large coefficients may be insignificant. An alternative representation of the effect of X on Y is the so-called impulse response function. By successive substitution, the vector autoregressive equations can be transformed into moving average representations expressing the current innovation in terms of lagged innovations.[16/] The response of the system to a set of shocks can then be examined. The resulting movement over time is labeled an impulse response function as the growth rate of Y is decomposed into dependence on shocks to the growth rates of all variables in the system. The impulse response functions for the SL results are presented in charts 1 to 4, where, for example, chart 1 plots the response over time of energy prices to innovations in each of the four series. Similarly, charts 5 to 8 contain the impulse response functions for the FPE results.

There are a number of interesting results based on the impulse response functions. First, compare the SL and the FPE based results. The patterns of responses are very similar although the SL responses generally dampen more rapidly. In terms of the individual responses, money growth appears to be most influenced by energy prices with the contractionary effect of higher oil prices apparently offset in part by expansionary monetary policy. In addition, there is also some indication of countercyclical monetary policy. Increases in output growth and inflation appear to produce short-run decreases in money growth.

Energy prices appear closest to being exogenous, responding only in the short run to inflation. Note that this result suggests that in the short run higher inflation results in both higher real and relative energy prices.

Output growth responds positively to money growth in the short run but over a longer time frame there are offsetting negative responses. This result is consistent with the conventional expectations-augmented Phillips curve analysis. Money growth can buy additional output growth only in the short run. Misperceptions about money growth and inflation are soon eliminated. Output growth is also apparently adversely affected in the short run by higher energy prices.

The impulse response functions for inflation suggests that money growth lowers inflation in the very short run but increases inflation substantially over longer periods. In the FPE based equation the null hypothesis that money growth increases inflation on a one-to-one basis in the long run could not be rejected ($t = 1.38$). Output growth also apparently increases inflation. Energy prices increase inflation in the short run with offsetting decreases in the long run based on the FPE results, while the VAR results do not suggest any long-run offsets. For the FPE results the null hypothesis that the sum of the energy coefficients in the inflation equation equals zero could not be rejected ($t = 1.05$).[17/] It should be noted that the testable restrictions of the assumption that the real wage is constant from equations (15) and (17)--dp/dm = 1 and dy/dm = 0--cannot be rejected in the long run based on the FPE results.

The last set of results is based on the variance decompositions (VDCs) which are also based on a moving average representation of the vector autoregressions. The VDCs decompose variation in the system into components due to variation in the shocks. For example, what percent of the variation in inflation is due to variation in money growth shocks? The SL results are presented in table 3, while the FPE results are in table 4. Two alternate decompositions are used in both cases to examine whether the results are sensitive to the ordering of the variables. Again, there are a number of noteworthy results.

First, compare the left-hand side decomposition for the SL and FPE results (with the assumed ordering M first and E last). In general, the SL VDC places more weight on the inflation variable than does the FPE VDC, especially in the inflation equation. This result should not be surprising since the FPE inflation equation includes much longer lags and has substantially smaller innovations in the P series. Otherwise, there is rough agreement on the relative magnitudes of the effects with money growth and real energy prices both influencing all four variables.

Second, compare the two decompositions for the FPE results (similar conclusions arise when comparing the VAR decompositions). It should be apparent that when real energy prices are ordered first rather than last, the weight assigned to it by the VDC increases dramatically while the weight for money growth declines. The explanation for this change lies in the contemporaneous correlation between M and E. When M is ordered first, the weight is assigned to M, while it is assigned to E if E is ordered first. While it is well known that the ordering may influence the weight assigned to alternate shocks, the results here provide an example of the importance of the ordering. The first ordering suggests money growth shocks are the key influence on inflation variability. The second suggests real energy price shocks are the key. The first is more accurately interpreted as stating money growth shocks are the key to inflation variability if energy prices respond contemporaneously to money growth. Alternately, energy prices are the key if money growth responds contemporaneously to energy prices. At this point we are reluctant to assign a zero probability to either of these positions but view the situation as more likely instantaneous (within one quarter) bidirectional causality (correlation) between money growth shocks and real energy price shocks. Thus, while the two VDCs presented represent the extreme causality assumptions, the truth likely falls between the extremes. Unfortunately, the VDC procedure does not allow a ready generalization to an intermediate case.

IV. Conclusions

The results presented here can best be considered exploratory, given the caveats outlined above. The results suggest using caution when applying vector autoregressive analysis and imposing the same lag length on all variables. The results generated may omit significant variables at longer lags, possibly giving rise to misleading causality conclusions, impulse response functions and variance decompositions. The specific results developed above strongly suggest that both money growth and energy prices have influenced the world economy. Money growth apparently has a one-to-one impact on inflation in the long run, while real energy prices changes the rate of inflation only in the short run.

A number of avenues remain open for future research. For example, it can be argued that important variables have been excluded. Interest rates, investment, unemployment rates and fiscal policy variables have all been included in national level studies but are excluded here. At this point, data on these series are apparently unavailable. Furthermore, use of OECD industrial production as our output measure may exaggerate the impact of energy prices on output. Nevertheless, we feel that further study along these lines can lead to a better understanding of the behavior of the world economy.

The approach employed here represents a promising halfway house between the simple, univariate ARIMA models and the complex LINK-type models. More generally, we observe that the increasing integration of the world economy makes such an approach more appealing. To quote Walter Salant (1977), "A supranational approach forces the analyst to abandon the deeply ingrained habits of thought imposed from birth by the natural character of institutions and by the whole culture." While the global approach is not without limitations, the national approach also has drawbacks including omitted external influences. At this point, it appears reasonable to view the national and the global approaches as yielding complementary insights into the world economic system.

FOOTNOTES

1/ See Rasche and Tatom (1977), Mork and Hall (1980), Darby (1982), Bruno and Sachs (1982), Hamilton (1983) and Burbidge and Harrison (1984).

2/ See Klein (1978) or Hickman and Schleicher (1978).

3/ Hickman (1983) and Huntington (1983) present summaries of the behavior of seven LINK-type models in response to various oil price shocks. Meese and Rogoff (1983) present a more definitive study of the weakness of the foreign exchange rate models.

4/ Salant (1977) was an early advocate of this approach. An effort in the LINK tradition that comes close to aggregate global modeling is that of Adams and Marquez (1983). They link the three aggregate groupings of OECD, OPEC and LDC's into a single model.

5/ For example, see Duck, Parkin, Rose and Zis (1976); Gray, Ward and Zis (1976); and Genberg and Swoboda (1977).

6/ The aggregate supply and demand approach was first applied at the national level by Weintraub (1958) and later by Davidson and Smolensky (1964). Gordon (1975) was among the first to associate raw materials prices with aggregate supply at the national level. Beenstock and Dicks (1983) also model global aggregate supply and demand at the OECD level. Their theoretical model differs from ours as do their estimation results. While they fit their results into a simple monetarist framework, their results can also be explained by an equilibrium, aggregate supply and demand approach. Energy prices affect only aggregate supply directly, whereas money supply unequivocally affects only aggregate demand directly. Observed values reflect the interaction of supply and demand. Expectations enter the model through the feedbacks from prices and outputs onto money supply and energy prices. Bruno and Sachs (1985) present a detailed theoretical model of global aggregate supply and demand but their empirical work remains at the national level.

7/ Another approach to global modeling is to construct univariate ARIMA equations for all variables of interest. Llewellyn and Arai (1984) have done this for OECD GNP. However, they themselves note that such models poorly predict the effects of exogenous shocks of other variables, notably oil prices.

8/ Furthermore, other variables have entered the determination of the equilibrium, notably w and k. We shall not explicitly model the latter two variables. The wage rate and capital stock are not explicitly modeled and are omitted from the estimated equations due to the lack of data. This omission can be justified either by assuming that they were constant over the interval or that they were functions of the endogenous variables. The real wage, if not constant, is likely positively related to movements in real output. Changes in the capital stock are generally hypothesized to be related to changes or accelerations in output.

9/ In addition, while the Hotelling supply assumptions may apply to fossil fuel sources, they do not necessarily apply to other fuels.

10/ For example, see Rosser and Sheehan (1985).

11/ The so-called Norwegian model focuses directly on the impacts of goods traded in world markets where the law of one price must prevail. For further detail, see Aukrust (1977).

12/ Unlike Litterman, however, we do not impose Bayesian priors.

13/ The complete F test results are:

For lags 7 and 8, $\chi^2(32) = 32.15$; significance = .459.

For lags 5 and 6, $\chi^2(32) = 30.23$; significance = .556.

For lags at 4, $\chi^2(16) = 13.75$; significance = .618.

For lags at 3, $\chi^2(16) = 39.17$; significance = .001.

14/ Note that the term causality as used here does not measure "philosophical causality" but merely the statistical significance of changes in variable A directly preceding changes in variable B. Furthermore, this "statistical precedence" only refers to the direct single equation effects and not the indirect effects working through the other equations in the system of equations.

15/ This point has been made in a bivariate framework by Thornton and Batten (1985).

16/ See Leiderman (1984) for more detail. See also Angeloni (1985) and Kling (1985) for a brief but cogent analysis of the debate concerning the use of impulse response functions and variance decompositions based on VAR analysis.

17/ A further comment is in order at this point on the level of aggregation. While it may be possible, for example, to aggregate inflation and output growth, it may be less reasonable to aggregate money growth. Consider a number of economies all characterized by many buyers and sellers. The decision functions of those economic agents will not all be the same, but economists routinely aggregate nonetheless. The implicit assumption is that we can focus on the representative agent. If individuals behave similarly across countries, then aggregating across countries represents no further simplification, and the aggregate demand and supply curves are effectively unchanged from their national counterparts. In contrast, monetary policy reaction functions may not be so readily aggregated. If monetary policymakers across countries have substantially different reaction functions, then these reaction functions could not be aggregated. While a law of large numbers may be invoked for supply and demand decisions for goods and services, it cannot be readily used to aggregate money supplies.

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Table 1
SL Results--Three Lags of All Variables
Marginal Significance Levels of Multivariate Causality Tests

Independent variables	Dependent variables			
	M	P	Q	E
M	.036*	.155	.625	.708
P	.506	.003*	.378	.008*
Q	.227	.199	.003*	.222
E	.017*	.036*	.016*	.080*

Table 2
FPE Results

Lag Lengths Selected

Independent variables	Dependent variables M	P	Q	E
M	8	4	0	0
P	8	8	0	2
Q	7	8	2	0
E	8	4	4	0

Results of Overfitting and Underfitting Tests 1/

Equation	Deleting (lags)	Significance	Equation	Adding (lags)	Significance
M	M(1-8)	.000	M	M(9-12)	.214
	E(1-8)	.000		E(9-12)	.138
	Q(1-7)	.000		Q(8)	.117
	P(1-8)	.000		P(9-12)	.354
P	P(1-8)	.000	P	P(9-12)	.349
	M(1-4)	.000		M(5-8)	.289
	E(1-4)	.000		E(5-8)	.510
	Q(1-8)	.006		Q(9-12)	.533
Q	Q(1-2)	.000	Q	Q(3-8)	.141
	E(1-4)	.000		E(5-8)	.431
				P(1-4)	.494
				M(1-4)	.870
E	P(1-2)	.000	E	P(3-8)	.632
				E(1-4)	.474
				Q(1-4)	.323
				M(1-4)	.393

1/ The overfitting results including 12 lags began in 1974:3.

Table 3
Variance Decompositions--SL 1/

After k quarters	Decomposition of variance in M				Decomposition of variance in M			
	M	Q	P	E	E	Q	P	M
1	100	0	0	0	14	0	7	78
4	75	7	4	14	33	10	7	50
8	66	9	8	16	40	13	6	41
12	65	10	9	16	40	13	6	40
24	65	10	9	16	40	13	7	40
	Decomposition of variance in Q				Decomposition of variance in Q			
	M	Q	P	E	E	Q	P	M
1	0	100	0	0	1	99	0	0
4	5	83	6	6	12	83	3	2
8	10	49	24	17	42	52	5	2
12	14	47	23	16	40	49	6	4
24	14	45	24	16	40	48	8	5
	Decomposition of variance in P				Decomposition of variance in P			
	M	Q	P	E	E	Q	P	M
1	0	0	99	0	25	1	73	0
4	3	4	79	15	51	6	41	2
8	5	6	72	12	42	9	42	8
12	9	6	69	11	37	8	44	11
24	17	5	68	10	34	7	46	13
	Decomposition of variance in E				Decomposition of variance in E			
	M	Q	P	E	E	Q	P	M
1	14	1	27	57	100	0	0	0
4	16	6	39	39	80	8	10	2
8	17	7	39	37	75	9	12	4
12	18	7	39	36	73	9	13	5
24	19	7	39	35	71	9	14	6

1/ Totals may not add to 100 due to rounding.

Table 4
Variance Decomposition Based on FPE Results 1/

After k quarters	Decomposition of variance in M				Decomposition of variance in M			
	M	Q	P	E	E	Q	P	M
1	100	0	0	0	47	2	11	41
4	59	12	6	22	71	12	4	13
8	46	20	6	28	65	19	7	9
12	45	18	6	32	68	17	7	8
24	42	17	6	35	68	17	8	7

	Decomposition of variance in Q				Decomposition of variance in Q			
	M	Q	P	E	E	Q	P	M
1	5	95	0	0	1	99	0	0
4	12	77	1	10	20	18	0	0
8	26	51	6	16	45	53	2	0
12	26	51	6	16	45	53	2	0
24	28	48	6	18	47	50	3	1

	Decomposition of variance in P				Decomposition of variance in P			
	M	Q	P	E	E	Q	P	M
1	47	1	54	0	43	0	57	0
4	37	22	26	15	52	21	23	4
8	34	33	20	12	42	35	19	3
12	35	31	14	19	49	34	14	2
24	40	24	9	27	61	27	10	2

	Decomposition of variance in E				Decomposition of variance in E			
	M	Q	P	E	E	Q	P	M
1	47	0	6	47	100	0	0	0
4	46	2	12	40	89	2	7	1
8	43	7	12	38	82	8	8	2
12	42	8	12	39	81	9	9	2
24	41	9	11	39	78	10	10	2

1/ Totals may not add to 100 due to rounding.

Chart 1
Responses of M

```
   -.0016          0                               .0039
     ++++++++++++++++++++++++++++++++++++++++++++++++++
 1   +             &                                  M QPE
 2   +         PQ  +M E                               +
 3   +   QE     P  +                  M               +
 4   E     PQ      +         M                        +
 5   +   EQP       +           M                      +
 6   +         Q  E+ PM                               +
 7   +           M Q     &                            + PE
 8   +        M    +Q   &                             + PE
 9   +           M + QEP                              +
10   +            M+E QP                              +
11   +             M E&                               + QP
12   +             +M&P                               + QE
13   +             +&P                                + MQE
14   +             &PM                                + QE
15   +            EQP M                               +
16   +            EQPM                                +
17   +             &PM                                + QE
18   +             &&                                 + QE  MP
19   +             &&                                 + QE  MP
20   +             &&                                 + QE  MP
21   +             Q&                                 + MPE
22   +             Q&                                 + MPE
23   +             +&                                 + MQPE
24   +             &&                                 + QE  MP
     ++++++++++++++++++++++++++++++++++++++++++++++++++
```

Chart 2
Responses of Q

```
   -.0061                0                           .0092
     +++++++++++++++++++++++++++++++++++++++++++++++++++
  1  +                  M&                             Q PE
  2  +             P     ME               Q            +
  3  +                  P+&  E                         + MQ
  4  +           EP  Q   +       M                     +
  5  P    E    Q         +        M                    +
  6  +     &  Q          +         M                   + PE
  7  +             &E    + M                           + QP
  8  +              M&   +E                            + QP
  9  +            M   QP + E                           +
 10  +             M    PQE                            +
 11  +             M   P +&                            + QE
 12  +              M  P +&                            + QE
 13  +               M  P+&                            + QE
 14  +                MP EQ                            +
 15  +                PMEQ                             +
 16  +                PE&+                             + MQ
 17  +                PE&+                             + MQ
 18  +                P &+                             + MQE
 19, +                PM&+                             + QE
 20  +                PM&+                             + QE
 21  +                 &&+                             + MP  QE
 22  +                 &&+                             + MP  QE
 23  +                 & &                             + MP  QE
 24  +                 & &                             + MP  QE
     +++++++++++++++++++++++++++++++++++++++++++++++++++
```

Chart 3
Responses of P

```
   -.0009      0                                     .0036
      ++++++++++++++++++++++++++++++++++++++++++++++++++
  1   +        E MQ                                    P
  2   M        +       Q         E      P              +
  3   +    M   +    Q   E      P                       +
  4   +        +   M   Q     E           P             +
  5   +        +  M     E    Q         P               +
  6   +        +     &     P                           + MQE
  7   +       E+ Q          &                          + MP
  8   +       E+ Q        P  M                         +
  9   +      E Q       P   M                           +
 10   +      Q E        P  M                           +
 11   +        &         &                             + QE MP
 12   +        QE      MP                              +
 13   +        QE     M P                              +
 14   +        +QE   M   P                             +
 15   +        + &  M   P                              + QE
 16   +        +QE  M   P                              +
 17   +        + &  M   P                              + QE
 18   +        + &  M  P                               + QE
 19   +        +&    MP                                + QE
 20   +        +&   M P                                + QE
 21   +        +&   M P                                + QE
 22   +        +&   MP                                 + QE
 23   +        +&   MP                                 + QE
 24   +        +&  M P                                 + QE
      ++++++++++++++++++++++++++++++++++++++++++++++++++
```

Chart 4
Responses of E

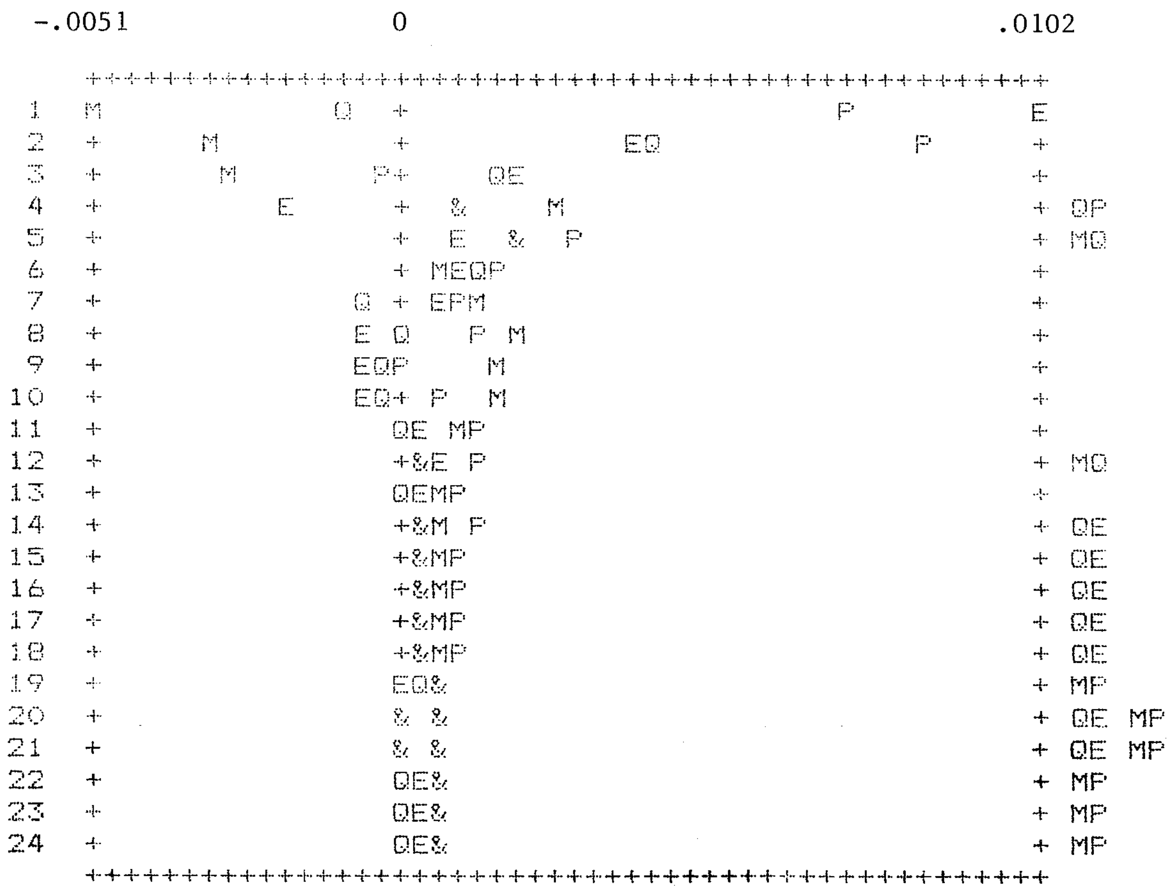

Chart 5
Responses of M

```
  -.0020                0                           .0030

     ++++++++++++++++++++++++++++++++++++++++++++++++++
  1  +                  &                         M   + QPE
  2  +    Q    P   M    + E                           +
  3  E     Q      P     +                             M
  4  +E         P       +   Q            M            +
  5  +      &      P    +           M                 + QE
  6  +          E       + P M   Q                     +
  7  +       Q E      M +     P                       +
  8  +          M     P +            Q  E             +
  9  +          E       +QP          M                +
 10  +         M        +  QP          E              +
 11  +   E       Q    P +        M                    +
 12  +            P    E+    Q  M                     +
 13  +         E        +  PQ    M                    +
 14  +                P ME  Q                         +
 15  +      E           +Q  P   M                     +
 16  +               M P+     &                       + QE
 17  +            E     P   Q   M                     +
 18  +               P  +E  MQ                        +
 19  +         E      Q +  P  M                       +
 20  +              P   E MQ                          +
 21  +        E         P  Q        M                 +
 22  +                 P+M E  Q                       +
 23  +         E       Q+ P  M                        +
 24  +                P +& Q                          + ME
```

Chart 6
Responses of Q

```
 -.0044             0                               .0093

    +++++++++++++++++++++++++++++++++++++++++++++++++++
 1  +               &      M                          Q PE
 2  +             EP+     M               Q           +
 3  +              MP E Q                             +
 4  + E         &   +           M                     + QP
 5  E   P      Q    +                   M             +
 6  +      E P   Q  +              M                  +
 7  +            QE PM                                +
 8  +         Q  & P+                                 + ME
 9  +            P &+E                                + MQ
10  +              M+&E                               + QP
11  +          M Q  +P  E                             +
12  +        Q  M   &                                 + PE
13  +           MQ  P   E                             +
14  +             M&+   E                             + QP
15  +            MQP+  E                              +
16  +          QM   E P                               +
17  +         M  Q  P  E                              +
18  +              &Q E                               + MP
19  +           M   QP  E                             +
20  +          M Q  + &                               + PE
21  +           MQ P+ E                               +
22  +              &PE                                + MQ
23  +            M  &  E                              + QP
24  +            QM EP                                +
    +++++++++++++++++++++++++++++++++++++++++++++++++++
```

Chart 7
Responses of P

```
 -.0019                     0                       .0020

    +++++++++++++++++++++++++++++++++++++++++++++++++++
 1  M                       E   Q                     P
 2  +M                      +    QP  E                +
 3  +                     M +   P     E     Q         +
 4  +              M        +             P    E  Q   +
 5  +               M   E  Q+          P              +
 6  +                   PE  +  M  Q                   +
 7  +              E        + P               M   Q   +
 8  +                     & +         M           Q   + PE
 9  +E                      +P     Q          M       +
10  +           E           +P        M  Q            +
11  +         E           P +               QM        +
12  +               PE      +                   QM    +
13  +     E                 +  P    Q           M     +
14  +             E       P +    Q M                  +
15  +  E             P      +      Q                M +
16  +               E P     +               Q   M     +
17  +       E               +P     Q        M         +
18  +           E       P   +   Q     M               +
19  +       E              P+    Q           M        +
20  +                  P  E +          M Q            +
21  +          E           P+    Q           M        +
22  +                E     P+  Q   M                  +
23  +          E            PQ         M              +
24  +                 P E   +        Q M              +
    +++++++++++++++++++++++++++++++++++++++++++++++++++
```

Chart 8
Responses of E

```
  -.0108                      0                      .0108

     +++++++++++++++++++++++++++++++++++++++++++++++++++
  1  M                        +Q       P               E
  2  +              M         EQ        P              +
  3  +                    &   Q  E                     + MP
  4  +                        PE   &                   + MQ
  5  +                     M  +    &                   + QPE
  6  +                  &   M +P                       + QE
  7  +                   P   E+ QM                     +
  8  +                     E  +P    MQ                 +
  9  +                      PM+E Q                     +
 10  +               E     Q  P    M                   +
 11  +                        M& Q                      + PE
 12  +                     E P+ QM                     +
 13  +                     P  E  &                     + MQ
 14  +                  E   Q + MP                     +
 15  +                       &QE                        + MP
 16  +                  E   P +Q      M                +
 17  +                        P+&   Q                   + ME
 18  +                    E  Q +&                      + MP
 19  +                       P&M                        + QE
 20  +                     E   PQ M                    +
 21  +                       P M EQ                    +
 22  +                    E   QP  M                    +
 23  +                        &E                        + MQP
 24  +                     E  QP M                     +
     +++++++++++++++++++++++++++++++++++++++++++++++++++
```