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STOCK PRICES, INFLATION AND REAL ACTIVITY: A TEST OF THE FAMA HYPOTHESIS, 1920-84

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1. Introduction

Orthodox theory suggests that stock prices move positively with inflation since stocks are claims to real assets and the real returns on such assets, in theory, are independent of inflation. Thus, stocks are viewed as a hedge against inflation [Cagan (1974), Lintner (1973)]. The preponderance of empirical evidence collected from the past few decades suggests otherwise. In fact, studies by (to name a few) Bodie (1976), Jaffe and Mandelker (1976), Nelson (1976), Fama and Schwert (1977), Schwert (1981), Solnik (1983) and Gultekin (1983) all find a significant negative relationship between stock prices and inflation in the United States and other countries.

In light of this empirical evidence, Fama (1981) claims that the observed negative relation between stock prices and inflation is the consequence of a strong negative link between inflation and real activity on one hand, and the strong positive correlation between stock prices and current and future real activity on the other.[1/] In his view, inflation acts as a proxy for real activity. Thus, including a measure of current and future real activity in an otherwise standard stock price-inflation regression should render the effect of inflation insignificant.

Fama's evidence is mixed. The negative coefficient found for expected inflation turns insignificant in the presence of output variables only if base money growth also enters the regression. Unexpected inflation always is negatively related to stock prices. Pearce (1984) also tests Fama's model, using expected stock price, inflation and output growth data taken from the Livingston semiannual survey. Pearce's results indicate that while the impact of expected inflation varies across his 1954-80 sample period, inclusion of expected real output growth does not render unexpected inflation insignificant. More importantly, however, is his finding of a <u>positive</u> response of expected stock price changes to expected inflation. More recently, Coate and Vanderhoff (1986) test Fama's conjecture by regressing actual real returns on stocks against expected inflation and expected output, the latter variables derived from the ASA-NBER quarterly survey. They conclude that the finding of a negative stock price-inflation relation is spurious, supporting Fama's theoretical argument that inflation acts merely to proxy future real output.

In this paper, we approach the issue in a somewhat different manner. Since orthodox theory originally found support from data of the 1920s and 1930s and rejections of the data have come from post-WWII studies, is there a temporal aspect to the problem that has gone largely untested? To be more specific, our purpose here is to examine the temporal stability of the stock price-inflation relationship over the period 1920-84, testing the strength of Fama's hypothesis across this broad period. Thus, we address the possibility that the current negative effect of unexpected inflation on stock prices is due to the effects of stagflation and that findings of a positive effect of unexpected inflation for the pre-war era also reflect a confounding of the inflation-output relationship. Put differently, is the positive inflation-stock price relationship found for earlier decades also a spurious result due to the omission of future real output growth?

The format of the paper is as follows. The following section discusses the relationship to be tested, presents the data and basic OLS results. Also included is a brief discussion of the procedure used to generate unexpected inflation. Section 3 reexamines the issue by incorporating measures of current and future real output growth into the test equations. Anticipating that discussion, our results suggest that incorporating output measures renders the unexpected inflation measure statistically insignificant across the six decades studied, while the impact of output growth varies in a distinct and important manner over the sample. We present our conclusions in section 4.

2. Model, Data and Empirical Results

The relationship between stock prices and inflation has been tested in many different ways. In the tests reported here, we confine ourselves to the relation between the quarterly logarithmic growth rate of Standard and Poor's Index (expressed in annual terms)--RS--and the unexpected component of the annualized logarithmic growth rate of the quarterly CPI.[2/]

2.1. Measuring Unexpected Inflation

Because we use a measure of unexpected inflation, it is necessary to first calculate an expected series. In this study we use the Multi-State Kalman Filter (MSKF) model that allows for both temporary and permanent changes in inflation and for temporary shocks to the price level.[3/] The structural equations describing this model are:

(1) $p_t = m_t + e_{1,t}$,

(2) $m_t = m_{t-1} + b_t + e_{2,t}$, and

(3) $b_t = b_{t-1} + e_{3,t}$,

where $e_{1,t}$, $e_{2,t}$ and $e_{3,t}$ are white noise shocks with mean zero and respective variances v_1, v_2 and v_3.

They are independently, normally distributed. The term p_t represents the logarithmic price level, m_t and b_t are the permanent level and growth rate of p_t. Estimates of m_t and b_t conditional on all information up to and including period t are used to generate a forecast for the next period. Observation p_{t+1} then allows us to estimate how much of the forecast error should be attributed to i) $e_{1,t+1}$, representing only temporary noise to the price level, implying that the structural parameters m and b should not be changed because of the current forecast error; ii) $e_{2,t+1}$, indicating that the permanent level has changed and needs adjustment; and iii) $e_{3,t+1}$, signaling a change in the growth of inflation that is permanent and should be incorporated in the new estimate of b. A useful characteristic of this procedure is that the ex ante nature of the expectations generating process is retained: forecasts are always conditional on past realizations of p_t only.

From equation 1 to 3, period t's expected inflation conditional on information known in period t-1 equals:

$$(4) \quad \hat{p}^e = (\hat{m}_{t-1} + \hat{b}_{t-1} - p_{t-1})400 ,$$

where $\hat{m}_{t-1}$ and $\hat{b}_{t-1}$ are the conditional estimates of m and b. Unexpected inflation, denoted $\hat{p}_t^{ue}$, is simply the difference between $\hat{p}_t$ and $\hat{p}_t^e$.

2.2. OLS Results

To examine the link between stock prices (RS) and unexpected inflation ($\hat{P}^{UE}$), we estimate the regression:

$$(5) \quad RS_t = \alpha_0 + \beta_1 \hat{P}_t^{UE} + \varepsilon_t$$

for the full sample period (I/1920–IV/1984) and two subperiods. Because many possible sample breaks are available, we have chosen to simply break the sample into two periods, I/1920–IV/1949 and I/1950–IV/1984. Equation 5 is estimated using OLS. Allowing for the possibility of heteroskedasticity, standard errors are calculated using White's (1980) procedure. To illustrate the effect of heteroskedasticity, we report standard t-statistics along with those based on White's correction. The regression results are presented in table 1. The full period result indicates that although the coefficient on unexpected inflation is positive, the estimated coefficient does not differ from zero at the 5 percent level of significance.

The subperiod results, while statistically weak, evidence the sign shift reported in previous studies. That is, unexpected inflation exerts a negative (and marginally significant) influence on stock prices during the post-WWII period, but not before.

2.3. Time-Varying Parameter Estimation

The immediate consequence of our preceding empirical result is to realize that standard regression analysis is inappropriate. We therefore modify our procedure by allowing the parameters to change over time without determining, a priori, specific break points or subperiods to be analyzed.

Equation (5) was estimated using the time-varying parameter approach (detailed in the appendix) for the 1920-84 period. The results are reported in the first tier of table 2. Testing this model against the null of no parameter variation (the OLS result), the calculated χ^2 statistic is 7.6, significant at the 1 percent level of significance. This result confirms our suspicion that one source of possible misspecification is the assumption of parameter constancy.

A useful aspect of the time-varying parameter estimation is the time series plot of the coefficient on unexpected inflation, reported in figure 1.[4/] The visual evidence supports the orthodox theory for the pre-1950 era, especially for the late 1920s: The path of the coefficient rises until 1933 and generally falls until 1974. The evidence in figure 1 is consistent with the OLS results in table 1.

To examine the relationship further, we applied the time-varying parameter estimation procedure to the two subperiods found in table 1. The results for the 1920-49 subperiod, reported in the middle tier of table 2, reveal that we cannot reject the null hypothesis of parameter constancy for this period. The calculated χ^2 statistic of 0.73 clearly falls below any reliable level of significance. Thus, for the 1920-49 period, the results reported in table 1 hold. The evidence for the recent 1950-84 period, found in the lower tier of table 2, rejects the null hypothesis of a stable coefficient on unexpected inflation. The time path for the coefficient, found in figure 2, confirms the oft reported negative effect found during the post-war period. It is interesting to note, however, that the coefficient is significantly negative only during the 1960s and early 1970s. In this regard, the evidence in figure 2 suggests that recent estimates are influenced largely by the relationship during roughly the 1966-74 period, one characterized by secularly rising unexpected rates of inflation.[5/] After this period, the plot indicates that the effect becomes smaller and not different from zero by the late 1970s.

The evidence obtained from the time-varying parameter tests reveals an unstable relationship between stock prices and unexpected inflation. The evidence presented in figures 1 and 2 indicates that stock prices are positively related to inflation during the pre-1950 period and, for the post-war period, evidence a negative relationship. In both instances, our evidence suggests that relatively brief periods during our sample may dominate the results, the late 1920s giving the positive relationship and the late 1960s-early 1970s producing the negative effect.

3. Tests of the Fama Hypothesis

Fama's hypothesis, to reiterate, states that the observed negative relation between unexpected inflation and stock price movements is caused by unexpected inflation merely acting as a proxy for future real activity. That is, during a period of stagflation, the negative output-inflation correlation dominates the positive inflation-stock price relationship. Thus, inclusion of expected real output in an equation of stock prices on inflation should render the inflation effect positive. As mentioned earlier, recent evidence by Coate and Vanderhoff (1986) suggests that once expected output is included, even unexpected inflation is rendered insignificant.

After some experimentation, the best fitting equation that incorporates Fama's hypothesis was found to be (for the full period):

$$(6)\quad RS_t = \underset{\substack{(.05)\\(.04)}}{-0.11} - \underset{\substack{(1.12)\\(0.98)}}{0.48}\hat{P}^{UE}_t + \underset{\substack{(7.25)\\(4.33)}}{0.82}\hat{y}_t + \underset{\substack{(2.15)\\(1.43)}}{0.39}\hat{\bar{y}}_t$$

$$\bar{R}^2 = 0.188 \qquad SE = 37.81 \qquad DW = 2.32$$

where $\hat{y}_t$ = current quarter growth rate of real output;

$\bar{\hat{y}}_t$ = average real output growth across periods t+1 through t+4.[6/]

The results indicate a strong, positive effect of current and future real growth on stock prices. The standard t-statistics (reported in the first set of parentheses) are quite large for both variables. When corrected for heteroskedasticity, however, the significance of future output falls, while contemporaneous output continues to exert a positive, statistically significant effect. An interesting aspect of the results in (6) is the impact on the inflation variable of including real output. Although the low significance renders the evidence suggestive at best, allowing for the effect of real output reduces the significance of unexpected inflation. More importantly, the estimated coefficient now shifts from positive to negative.

To further investigate the effect on the unexpected inflation coefficient over time from including the output measures, the time-varying parameter procedure was used to reestimate equation (6), first allowing only the coefficient on $\hat{P}^{UE}$ to vary, then allowing both output coefficients to vary. The outcome from the first experiment is:

$$(7) \quad RS_t = 0.21 + \hat{\beta}_t \hat{P}^{UE}_t + 0.78\hat{y}_t + 0.37\hat{\bar{y}}_t$$
$$\qquad (0.09) \qquad\qquad (7.09) \quad (2.06)$$

$$\hat{\sigma}^2_\varepsilon = 13.79 \qquad \hat{\sigma}^2_p = 7.02 \qquad \chi^2(1) = 3.0$$

The χ^2 statistic of 3.0 indicates that we cannot reject stability at the 5 percent level, although rejection is possible at the 10 percent level. The time path of $\hat{\beta}_t$ is illustrated in figure 3. The evidence there suggests that the positive impact found earlier disappears in the presence of output growth. Also, while unexpected inflation's negative effect again appears after WWII, the significance of the negative effect is dampened somewhat. Given the effect of current and future real growth, unexpected inflation's significant negative effect is confined to the early 1970s.

Allowing the output measures to vary produces the result:

$$(8) \quad RS_t = 2.52 - 0.34\, \hat{P}^{UE}_t + \gamma_t \hat{y}_t + \delta_t \hat{\bar{y}}_t$$
$$\qquad (1.09) \; (0.87)$$

$$\hat{\sigma}^2_e = 9.76 \qquad \hat{\sigma}^2_{\hat{y}} = 8.83 \quad \hat{\sigma}^2_{\bar{y}} = 1.13 \quad \chi^2(2) = 52.64$$

where the χ^2 statistic easily rejects the null hypothesis of parameter stability. Note also the continued insignificance of unexpected inflation once current and future real activity are accounted for.

The behavior of the output coefficients is reported in figures 4 and 5. Figure 4--plotting the effect of current output--shows a strong positive impact during the 1920s, peaking at the time of the Depression and falling to essentially zero from the mid-1930s through the remainder of the sample. In contrast, the time path for future output--reported in figure 5--indicates an insignificant effect prior to 1950 and, afterward, a significant, increasingly positive influence.

The contrasting behavior of the two output effects suggests the following explanation. Prior to WWII, output growth and stock prices were strongly correlated, but stock returns did not perform well as a leading indicator of future real activity. During the post-war period, however, future growth, not current growth, is highly correlated (positively) with stock returns, demonstrating the leading indicator function of stock prices, a notion that is supported by the evidence found in Huang and Kracaw (1984). Indeed, this conjecture is supported by further evidence (not reported) indicating that, based on applying the time varying parameter model to the subperiods, unexpected inflation and future growth have stable but insignificant coefficients over the 1920-49 period. The coefficient on current output growth, as expected from figure 4, is unstable, significantly positive during the 1920s and falling from about 1930 through 1936 when it becomes insignificantly different from zero. For the 1950-84 period, no significant variation is found for any coefficient. More importantly, however, is the fact that the coefficient on future real growth is significantly different from zero and positive. Thus, our results for the post-1950 period show that once future real output growth is included, the influence of unexpected inflation is reduced to zero.

4. Conclusion

The evidence presented in this paper corroborates the results presented in Fama (1981) and Coate and Vanderhoff (1986). We show that the recent reports of a negative stock price-unexpected inflation relation is due to the negative correlation between output and unexpected inflation. Accounting for current and future real activity via Fama's interpretation of a forward looking demand for money renders the estimated effect of unexpected inflation on stock prices insignificantly different from zero during the post-1950 period. While the recent period's evidence supports Fama's hypothesis, the results for the pre-1950 period indicate that earlier evidence for this period in support of the orthodox theory may be spurious. Our findings for this earlier period suggest a strong, positive contemporaneous correlation between current real activity and stock returns that dominate effects from inflation and future activity.

FOOTNOTES

1/ See Geske and Roll (1983) for an alternative explanation.

2/ We are aware of the criticisms leveled at the CPI. It is, however, a consistent time series across the sample period used here.

3/ The approach used here is more flexible in providing a time series of expected and unexpected inflation than an ARIMA model. The reason is that the MSKF approach allows new information to "update" the relative effects of permanent and transitory changes on the underlying structure generating observed inflation. Thus, shocks to inflation are decomposed according to expected persistence and effects on levels or growth rates. Examples of recent studies employing this procedure include Bomhoff (1982, 1983), Hafer, Hein and Kool (1983) and Meltzer (1984, 1985). For a detailed description of the estimation technique, see Harrison and Stevens (1971, 1976) and Kool (1983).

4/ See Anderson and Moore (1979) for a discussion of how such smoothed estimates are obtained.

5/ The notion that bursts of inflation were in large measure unexpected is borne out in numerous studies examining the accuracy of survey forecasts of inflation. The general result from these studies is that forecasters underpredicted the rise in prices during the mid-1970s and early 1980s. Moreover, it should be noted that many forecasting models also failed to capture the move in inflation. For a comparison of forecasts taken from different models, see Hafer and Hein (1985).

6/ The output measure used is industrial production. We follow Fama in the use of actual future real output. Experimentation with alternative measures (i.e., model based) was not successful.

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Table 1
Regression Results *
Equation Tested: $RS_t = \alpha_0 + \beta_1 \hat{P}_t^{UE} + \varepsilon_t$

Period	α_0	β_1	$\bar{R}^2$	SE	DW
I/1920-IV/1984	4.53 (1.74) (1.74)	0.59 (1.33) (0.95)	0.003	41.89	2.19
I/1920-IV/1949	2.39 (0.49) (0.49)	0.92 (1.51) (1.25)	0.011	53.31	2.38
I/1950-IV/1984	6.84 (2.87) (2.90)	-1.70 (2.05) (1.74)	0.022	28.14	1.82

* Absolute value of standard t-statistics reported in first row of parentheses. White (1980) heteroskedasticity corrected t-statistics reported in second row.

Table 2
Time-Varying Parameter Results
Equations Tested: $RS_t = \alpha_0 + \beta_t \hat{P}_t^{UE} + \varepsilon_t$

$$\beta_t = \beta_{t-1} + \mu_t$$

Period	α_0	$\hat{\sigma}_\varepsilon^2$	$\hat{\sigma}_\mu^2$	L	$\chi^2(1)$
I/1920-IV/1984	4.78 (1.89)	16.35	20.38	-223.14	7.6
I/1920-IV/1949	2.21 (0.46)	27.43	10.08	-69.87	0.73
I/1950-IV/1984	7.09 (3.06)	7.21	57.25	-173.64	5.68

FIG.1: RS=F(PUE) 1920-84

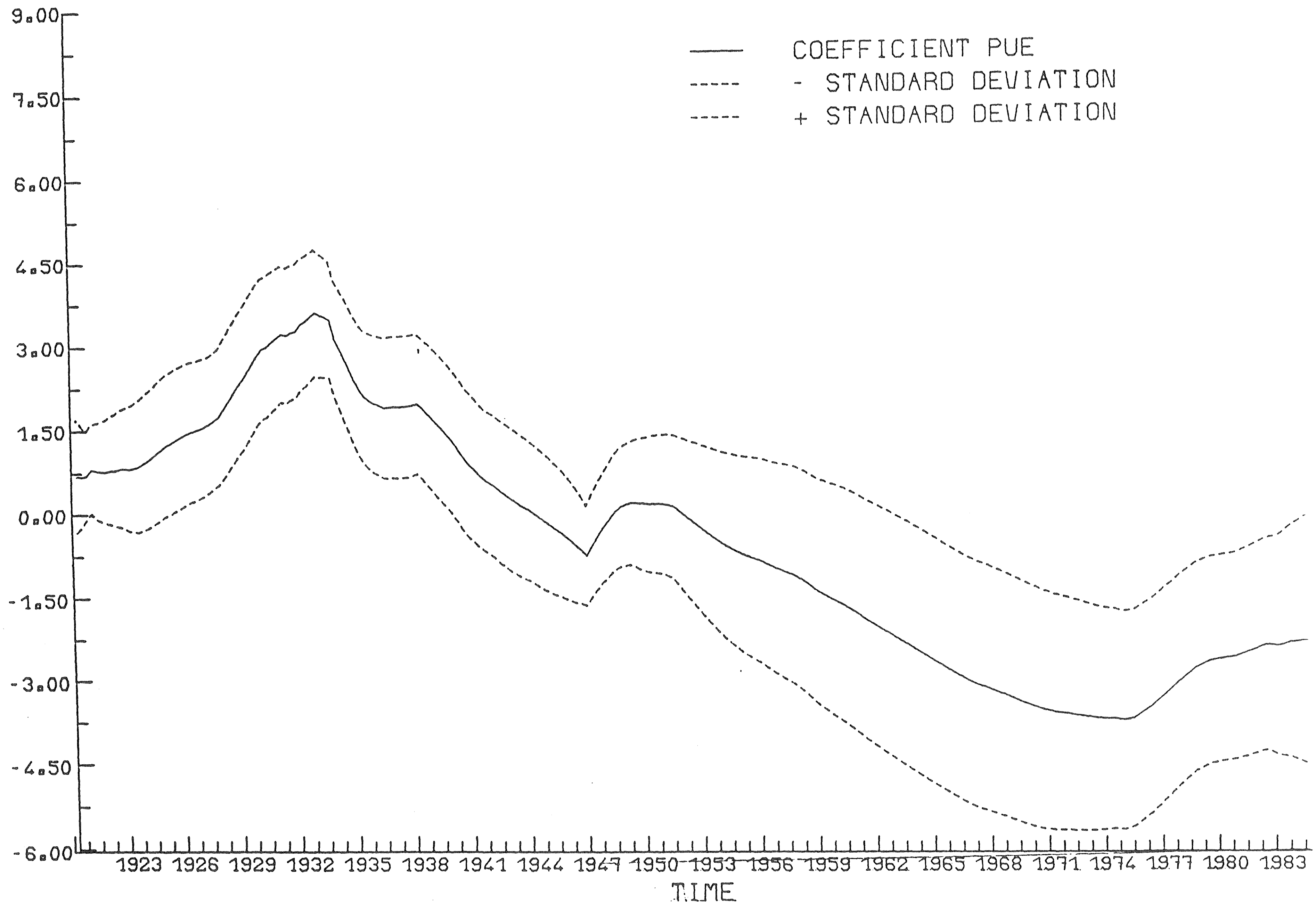


FIG.2: RS=F(PUE) 1950-84

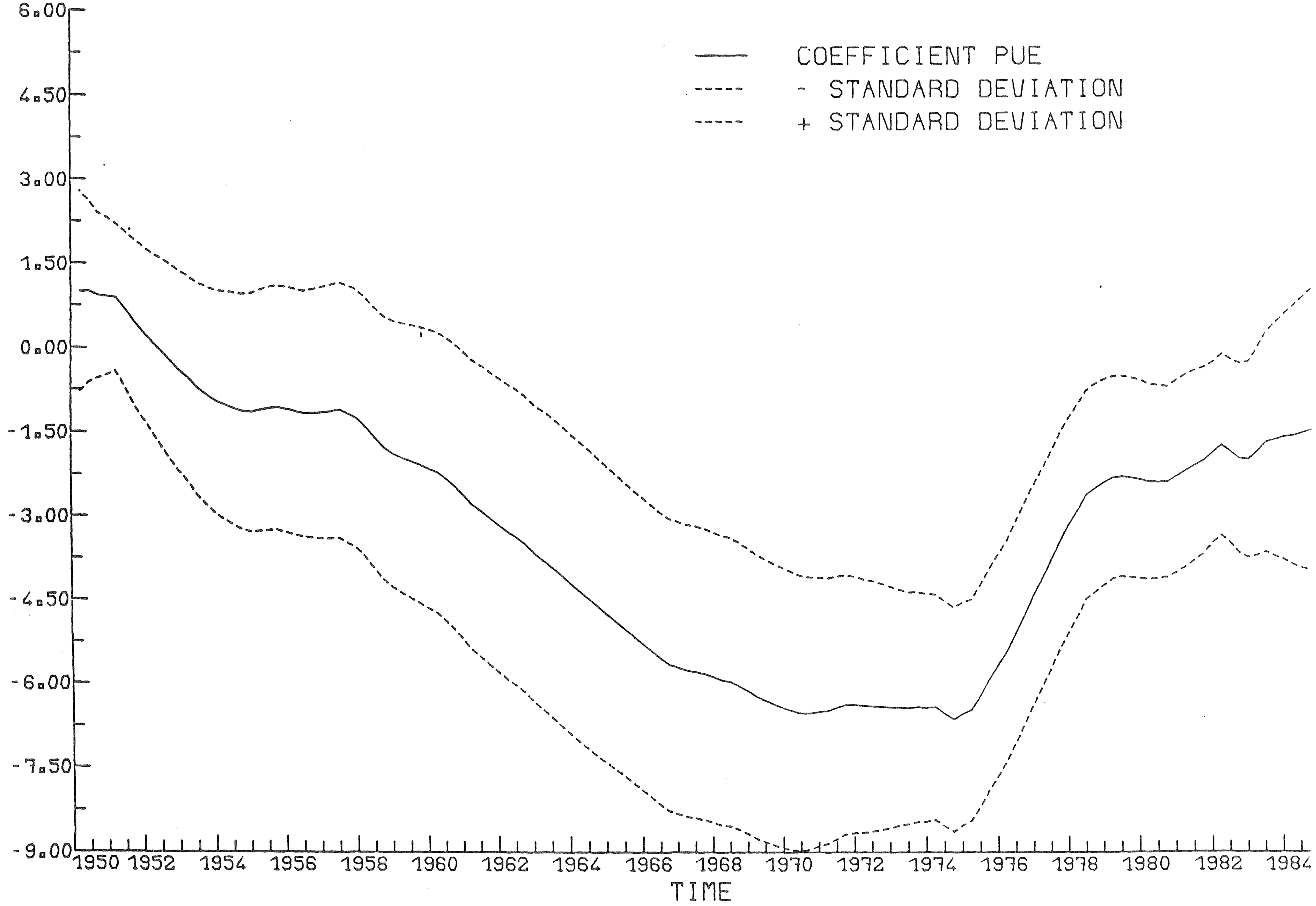


FIG.3: RS=F(PUE|Y,Y4)

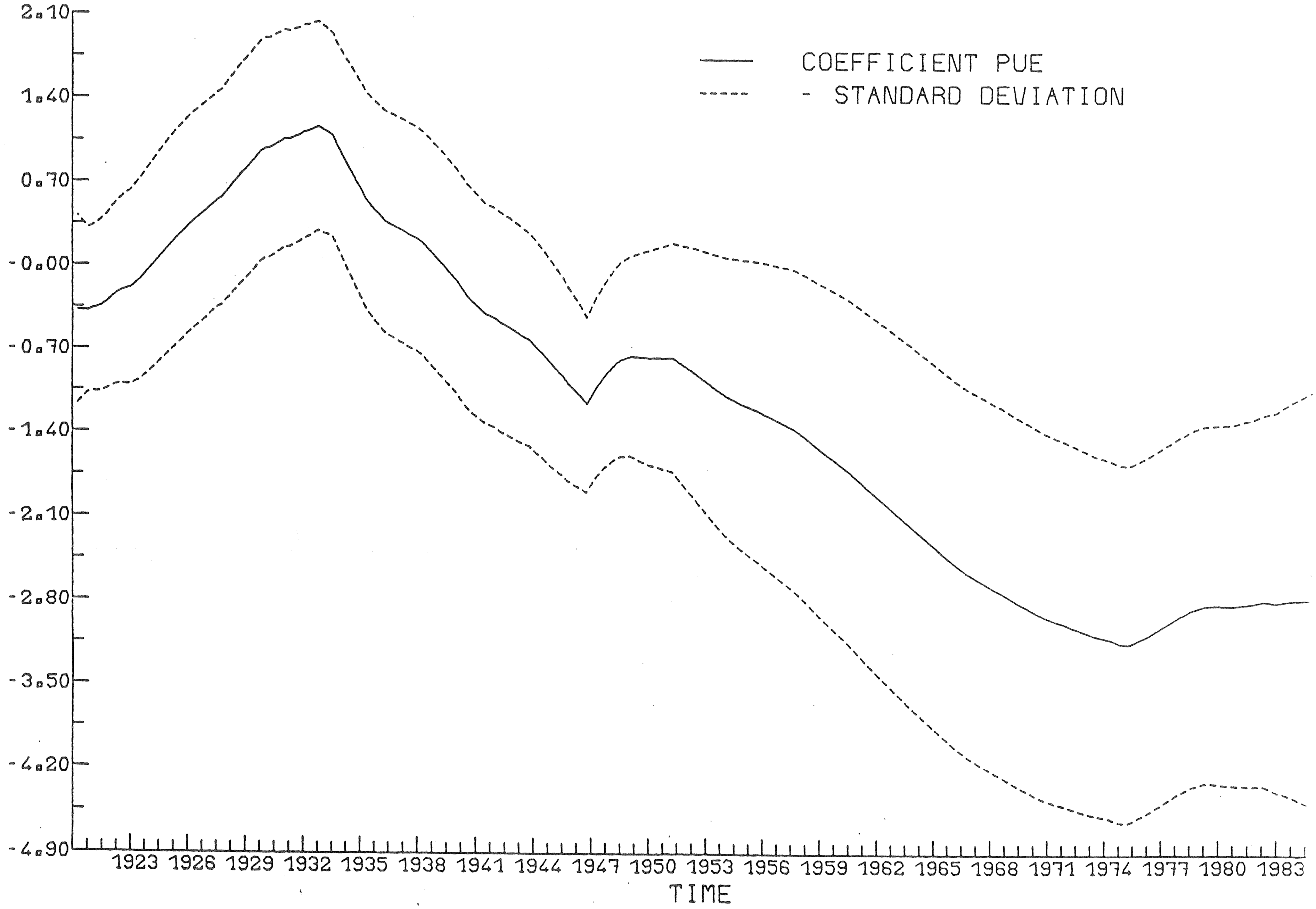


FIG.4: RS=F(Y,Y4|PUE)

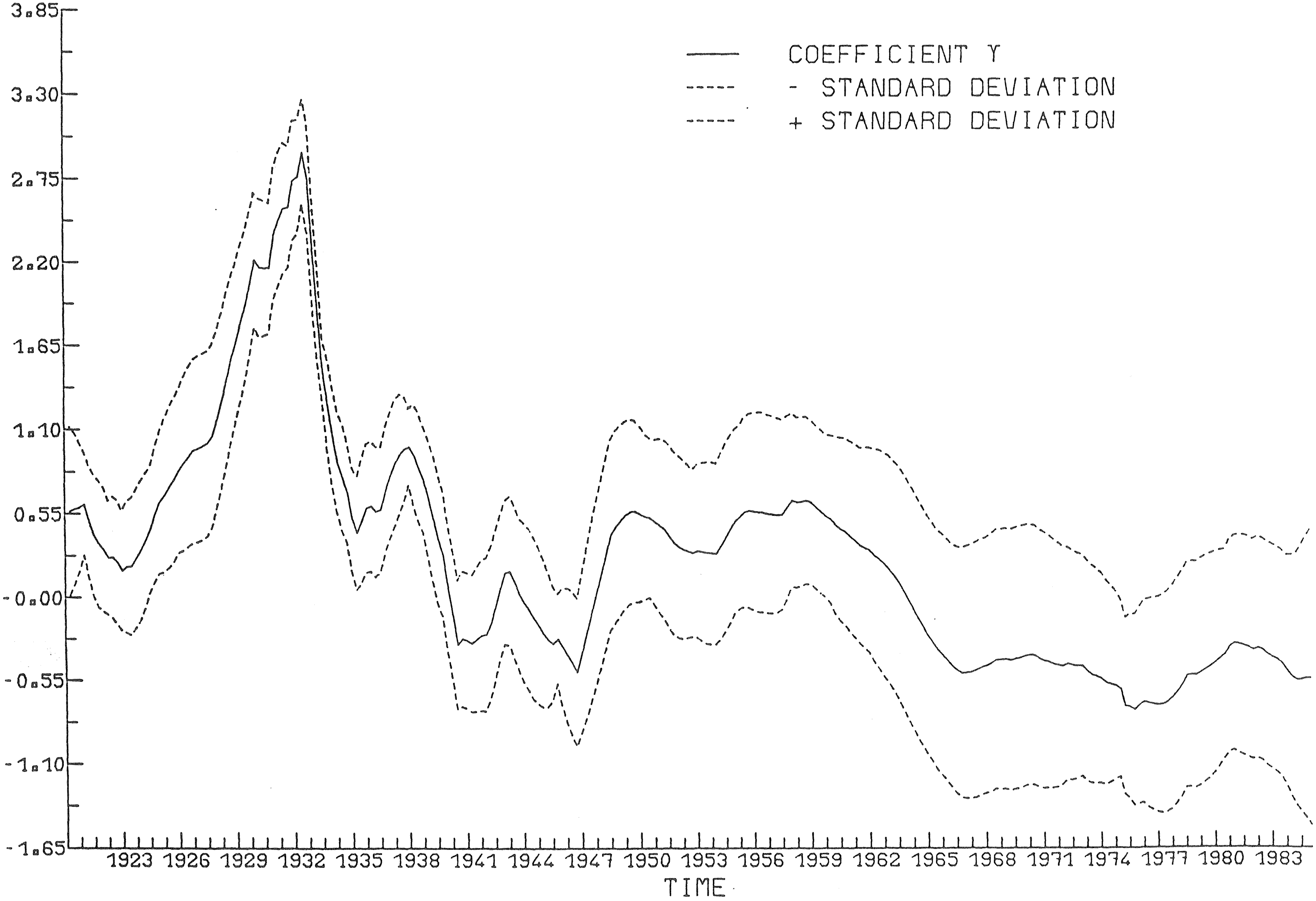
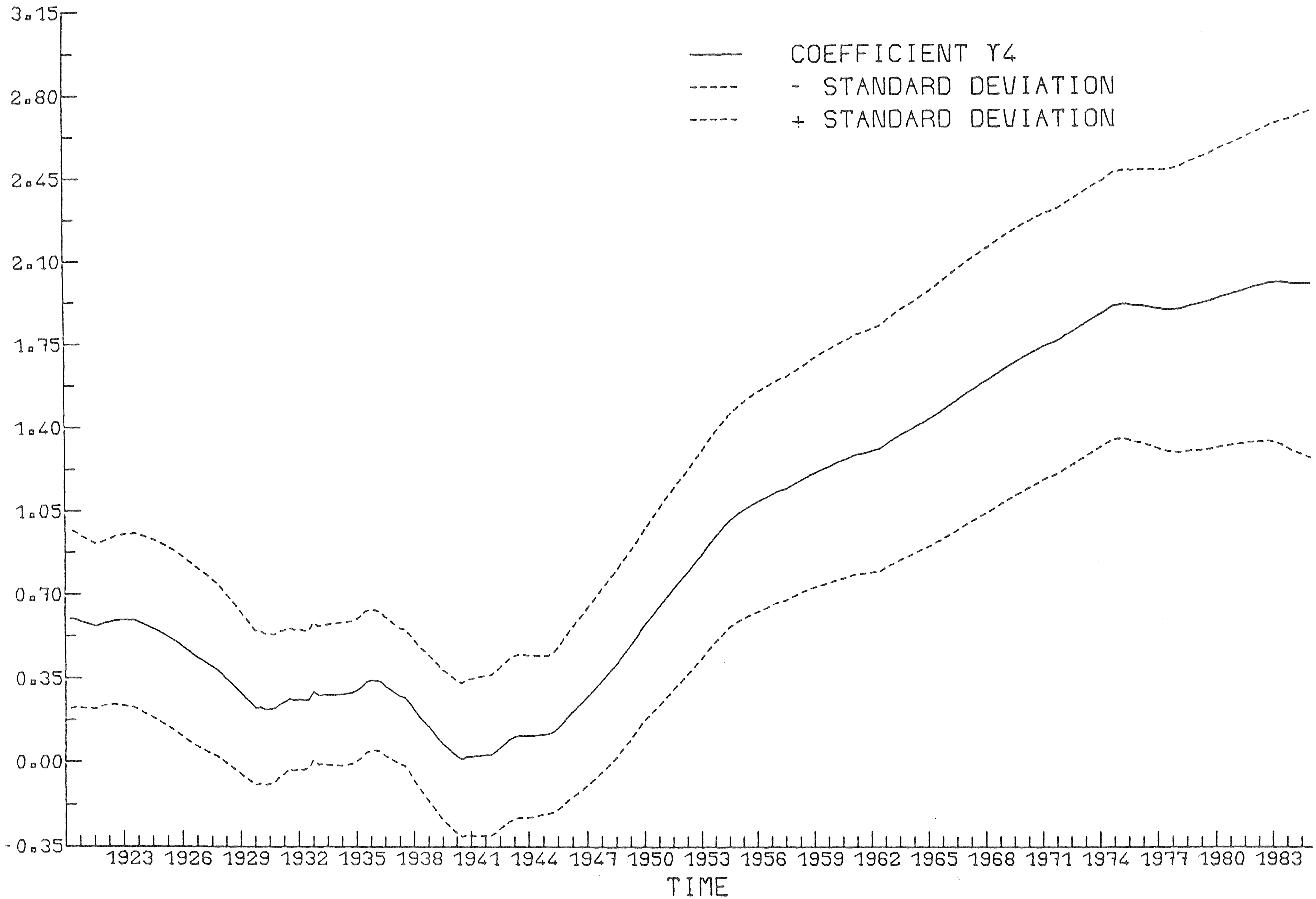


FIG.5: RS=F(Y,Y4|PUE)



Appendix

Time-Varying Parameter Estimation

The general regression problem takes the form:

$$(1)\quad y_t = x_t b_t + u_t \qquad u_t \sim N(0,\sigma^2)$$

$$(2)\quad b_t = b_{t-1} + v_t \qquad v_t \sim N(0,\sigma^2 P)$$

where x_t is the kx1 vector of explanatory variables, b_t is the k-dimensional parameter vector, u_t is the normally distributed scalar measurement error with mean zero and variance σ^2 and v_t is the vector of innovations driving the parameters. The elements of v_t are independently normally distributed with mean zero and covariance matrix $\sigma^2 P$, where P is a diagonal matrix. Though a more complex dynamic structure for b_t is possible, b_t is restricted to behave as a simple random walk in this study.

The model parameters of equations 1 and 2 to be estimated are σ^2 and the diagonal elements of the covariance matrix P. The log likelihood function to be maximized equals:

$$(3)\quad \log L = -0.5 \sum_{t=k+1}^{N} (e_t^2/\sigma^2 E_t) - 0.5 \sum_{t=k+1}^{N} \log(\sigma^2 E_t)$$

where e_t is the one period ahead forecast error $(y_t - x_t \hat{b}_{t|t-1})$, $\sigma^2 E_t$ is the forecast error variance and E_t is defined as $(1+x_t R x_t')$, where R is the estimation covariance matrix of $\hat{b}_{t|t-1}$. From the likelihood function, an analytic estimate of $\hat{\sigma}^2$ follows immediately:

$$(4) \quad \hat{\sigma}^2 = \sum_{t=k+1}^{N} (e_t^2/(N-k)E_t) \; .$$

The concentrated likelihood function to within a constant then is:

$$(5) \quad \log L^* = -(N-k)\log(\hat{\sigma}) - 0.5 \sum_{t=k+1}^{N} \log (E_t)$$

Because both $\hat{\sigma}_t^2$ and E are complex functions of the elements of P, numerical maximalization methods are necessary to calculate optimal values for P. We always start the recursions with $\hat{b}_0$ as zero, and R_0 as a large positive number multiplied by the identity matrix. After k recursions, where k is the number of explanatory variables, we start cumulating the likelihood function.