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A Portfolio Choice Model for Analyzing the Impacts of Government Loan and Guarantee Programs

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CANADA

A PORTFOLIO CHOICE MODEL FOR ANALYZING THE IMPACTS OF GOVERNMENT LOAN AND GUARANTEE PROGRAMS

This appendix develops the underlying portfolio choice model used in the text.[1/] A simple stylized economy is hypothesized consisting of four sectors--the non-financial private sector, financial intermediaries (called banks), the government and the Federal Reserve--and seven types of financial instruments--the monetary base (B), bank deposits (D), privately negotiated uninsured loans (PL), government guaranteed loans (GGL), government direct loans (GL), government securities (G) and titles to the capital stock (qK). The private non-financial sector has real liabilities of all PL*, GGL*, and GL* and real assets of B^{*h}, G^{*h}, and the entire stocks of D* and qK. Starred variables denote the real values of nominal magnitudes (e.g., PL* = PL/P where P is the price level), the superscript h refers to the holdings of the non-financial private sector, and q is the demand price for capital, K. Banks have real asset holdings of B^{*b}, G^{*b}, PL* and GGL*. They are the sole issuers of deposits. The Federal Reserve has real liabilities of $B^* = B^{*h} + B^{*b}$, and assets of G^{*f}. The government has real liabilities of G* and assets of GL*. It has implicit liabilities of the present value of the subsidies paid on GL* and GGL*.

In the next section, financial intermediaries are modeled and in the section following that the general portfolio equilibrium is characterized. The final two sections examine compensated increases in direct government loans and increases in government direct loans and guarantees separately.

Modeling Financial Intermediation

Costs to the representative bank can be decomposed into the costs of raising funds and the costs of default, processing, and monitoring loans. Funding costs are

$$\text{(A1)} \qquad TCF = R_d D^* + C^b(D^*, G^{*b}, B^{*b})$$

where $C^b(D^*, G^*, B^*)$ represents the costs of providing liquidity services to bank customers and R_d is the real rate of return on bank deposits. It is assumed that the partial derivatives of $C^b(\ )$ take the following signs:

$$\text{(A2)} \qquad C^b_D \geq 0 \geq C^b_G \geq C^b_B,$$

and that

$$\text{(A3)} \qquad C^b_{GD}, C^b_{BD} \leq 0 \leq C^b_{GB}, C^b_{BB}, C^b_{GG}$$

where C^b_X is the partial derivative of C^b with respect to the real value of X. Note that $-C^b_G$ and $-C^b_B$ can be interpreted as the "liquidity yield," or marginal productivity, of government securities and monetary base, respectively.[2/]

Total costs of handling loans by the bank are described by:

(A4) $TCL = \Delta_{\ell}\ PL^* + \Delta_g\ GGL^* - (\alpha\Delta_g + T)GGL^* + \theta(PL^*, GGL^*)$,

where $\Delta_{\ell}(\Delta^g)$ is the expected default rate on private (government guaranteed) loans, T represents any transfers per dollar of guaranteed loan the government provides the bank above that amount to cover actual defaults, $\theta(PL^*, GGL^*)$ represents other costs of handling the loan portfolio that are unrelated to default costs, and α is the proportion of default costs covered by government guarantees, $0 < \alpha < 1$. It is assumed that:

(A5) $\partial\theta/\partial PL^*,\ \partial\theta/\partial GGL^* \geq 0$

and that

(A6) $\partial^2\theta/\partial PL^*,\ \partial^2\theta/\partial GGL^{*2},\ \partial^2\theta/\partial GGL^*\partial PL^* \geq 0$.

Total revenue to the bank is

(A7) $TR^* = R_{\ell}PL^* + R_{gg}GGL^* + R_g G^{*b} + R_b B^{*b}$,

where R_i is the real rate of return on asset i. Thus, the bank decision problem can be characterized as:

(A8) $$\underset{\{D, G^b, B^b, PL, GGL,\}}{MAX}\ \Omega = TR^* - TCF^* - TCL^*$$

$$= (R_{\ell} - \Delta_{\ell})PL^* + [R_{gg} - (1-\alpha)\Delta_g + T]GGL$$

$$+ R_g G^{*b} + R_b B^{*b} - R_d D^* - C^b(D, B^b, G^b) - \theta(PL^*, GGL^*)$$

subject to the balance sheet identity,

(A9) $D^* = B^{*b} + G^{*b} + PL^* + GGL^*$

and to

(A10) $D^*,\ G^{*b},\ B^{*b},\ PL^*,\ GGL^* \geq 0.$

Assuming an interior solution, the first order conditions to this problem are:

(A11) (a) $\partial\Omega/\partial PL^* = R_\ell - \Delta_\ell - \partial\theta/\partial PL - \lambda \qquad = 0$

(b) $\partial\Omega/\partial GGL^* = R_{gg} - (1-\alpha)\Delta_g + T - \partial\theta/\partial GGL^* - \lambda \qquad = 0$

(c) $\partial\Omega/\partial G^* = R_g - C_G^b - \lambda \qquad = 0$

(d) $\partial\Omega/\partial D^* = -R_d - C_D^b + \lambda \qquad = 0$

(e) $\partial\Omega/\partial B^* = R_b - C_B^b - \lambda \qquad = 0$

where λ is the lagrangian attached to (A9); λ can be interpreted as the marginal cost of funds to the bank. Note that the net rates of return on private loans, $R_\ell - \Delta_\ell - \partial\theta/\partial PL^*$, and government guaranteed loans, $R_{gg} - (1-\alpha)\Delta_g + T - \partial\theta/\partial GGL$, are equalized.

To construct demand and supply functions for the banking industry, the following simplifying assumptions are made: (1) both $\theta(PL^*, GGL^*)$ and $C(D^*, G^{*b}, B^{*b})$ are linear homogeneous, (2) all firms in the banking industry are identical, and (3) the banking industry is competitive. These assumptions permit us to solve for R_d as a function of R_g, R_b, R_ℓ, $R_{gg} + T$ and α:[3/]

(A12) $R_d = d(R_g,\ R_b,\ R_\ell,\ R_{gg} + T_g,\ \alpha)$

where the partial derivatives of (A12) are all positive. Using (A12) and the homogeneity assumption implies that demands for $B*^b$ and $G*^b$ and supplies of credit by banks can be described by the functions

(A13) (a) $B*^b = b_b(R^b)D*$

(b) $G*^b = b_g(R^b)D*$

(c) $PL* = b_{p\ell}(R^b)D*$

(d) $GGL* = b_{gg}(R^b)D*$

where $R^b = (R_g, R_b, R_\ell, R_{gg} + T, \alpha)$, the vector of exogenous parameters the banking industry faces. The partial derivatives of (A13a) - (A13d) are all negative except for $\partial b_b/\partial R_b$, $\partial b_g/\partial R_g$, $\partial b_\ell/\partial R_\ell$, $\partial b_{gg}/\partial(R_{gg} + T)$ and $\partial b_{gg}/\partial\alpha$, which are positive.

Portfolio Equilibrium

Households are assumed to hold all the assets in the non-financial private sector, with firms serving the role of transforming labor and capital services into a composite consumption/capital good. Household excess demands depend upon aggregate real (tangible) wealth and interest rates on the various financial instruments. Real wealth for the private non-financial sector is

(A14) $V = qK + [B*^h+G*^h+D*-PL*-GGL*-GL*]$.

Making use of the bank balance sheet identity (A9), (A14) becomes

(A15) $V = qK + (B^{*}+G^{*p}-GL^{*})$

where $G^{*p} = G^{*h} + G^{*b}$.

The interest rates that influence household demands are R_g, R_b, R_d, R_ℓ, R_{gg}, $R_{g\ell}$ and R_k, where $R_{g\ell}$ is the rate of interest on government loans and R_k is the expected rate of return on titles to the capital stock and is defined by

(A16) $R_k = R^{*}/q$

where R* is the expected marginal product of physical capital. Households are assumed to be indifferent to which institution grants them a loan or whether or not it is guaranteed by the government. Rather the sole criterion is in terms of the rate charged. To reflect this assumption, it is supposed that R_{gg} and $R_{g\ell}$ enter the demand functions in the form γ, where γ is defined as

(A17) $\gamma = (R_\ell-R_{gg})\overline{GGL}^{*} + (R_\ell-R_{g\ell})\overline{GL}^{*}$.

$\overline{GGL}^{*}$ and $\overline{GL}^{*}$ may be taken as either the real value of actual government guaranteed and direct loans <u>if</u> the government chooses to set those levels <u>or</u> the maximum valve of direct and/or guaranteed loans mandated by Congress. In the latter case, the actual levels of government guaranteed and direct loans are endogenous.

Using (A12), household asset demands can then be described as functions of $R^h = (R_k, R_g, R_b, R_\ell, \gamma)$, and V. It is assumed that all demands are linear in V; the other arguments determine the proportion of household wealth held in each type of financial instrument. Combining household demands, bank demands and supplies (given by (A13)) and government supplies, the market equilibrium conditions are given by:

(A18) $a_k(R^h)V = qK$

(A19) $a_b(R^h)V + b_b(R^b)D^* = B^*$

(A20) $a_g(R^h)V + b_g(R^b)D^* = G^{*P}$

(A21) $a_d(R^h)V - D^* = 0$

(A22) $-a_\ell(R^h)V + b_{p\ell}(R^b)D^* + b_{gg}(R^b)D^* = -GL^*$

(A23) $-a_\ell(R^h)V + GGL^* + GL^* + PL^* = 0$

(A24) $b_{gg}(R^b)D^* = GGL^*$

To complete the system, government rules for interest rate settings on GGL and GL have to be made, as well as rules for the amounts the government chooses to supply of the assets under its control. For the interest rate on government direct loans, it will be assumed that

(A25) $R_{g\ell} = R_{gg}$.

For quantities, it will be assumed that

(A26) $B = \underline{B}$,

(A27) $G = \underline{G}_n + \underline{G}^f + GL$,

and

(A28) $GL = \underline{GL}$,

where $\underline{G}_n$ is the exogenous stock of net government debt held by the non-government sectors. The setting of R_{gg} and GGL is somewhat more complicated because the government has to devise mutually consistent rules for GGL, R_{gg}, T and α. Assume α is given and consider the setting of the other three. Alternatives that are feasible are:

(1) set T and GGL according to

(A29) $T = \underline{T}$

(A24') $GGL = \underline{GGL}$;

(2) Set R_{gg} and GGL according to

(A30) $R_{gg} = R_{\ell} - \underline{S}$

and (A24'), where $\underline{S}$ is the subsidy rate, as seen by borrowers, on government guaranteed loans;

(3) set R_{gg} and T according to (A30) and (A29); and

(4) set T according to (A29) and let R_{gg} and GGL be determined in the market.

If the first of these alternatives is used, it implies some rationing of government guaranteed loans to households. If the banking industry is competitive, however, this rationing will not generate excess profits since eligible borrowers can shop around for the best offer. In this case, (A24) and (A24') together determine the excess demand for GGL that serves as the proximate determinant of R_{gg}.

If the second alternative is chosen, then the transfer rate, T, is endogenous and is determined by:

(A29') $T = \underline{S} - \Delta^{\ell} - (1-\alpha)\Delta^{g}$

$+ \partial\theta/\partial GGL - \partial\theta/\partial PL = \underline{S} + \Psi(\alpha) + \partial\theta/\partial GGL - \partial\theta/\partial PL$

rather than by (A29). Further, (A24) is no longer relevant for the determination of R_{gg} and (A30) would replace it in the general equilibrium system. The logic of choosing this alternative would be that the government itself would choose the eligible borrowers and then serve as the agent for the borrower by auctioning off the right to lend to these individuals at the rate $R_{\ell} - \underline{S}$ to the banks. Those banks that require the lowest T would win the lending rights in the auction.

The third alternative means that, having designated the group eligible for government guaranteed loans and the rate on these loans, the government leaves to the banks and to households the issue of how these loans will be allocated and to whom. Since R_{gg} is fixed (relative to R_{ℓ}), some parties will generally be rationed. We shall suppose the short side of the market represents the actual amounts of GGL that will be issued. This will be described by (A24) if banks are on the short side and by

(A31) $-a_{gov}(R^h)V + GL^* = -GGL^*$,

if banks are on the long side of the market and where $a_{gov}(R^h)V$ is household demand for government direct and guaranteed loans.

For the final alternative, (A24) and (A31) constitute the market for government guaranteed loans, and serve as the proximate determinants of GGL and R_{gg}. For our purposes, the first alternative is the most illuminating, so we shall assume it to be the case in the following analysis.

The 17 equation system, (A12), and (A15) through (A29) and (A24') consists of 16 independent equations that can, in principle, be used to solve for the 16 independent variables, V, q, R_k, R_g, R_ℓ, R_{gg}, $R_{g\ell}$, R_d, γ, GGL, GL, G, PL, D, B and T. The exogenous variables are K, $\underline{G}_n$, $\underline{G}^f$, $\underline{GGL}$, $\underline{GL}$, $\underline{B}$, $\underline{T}$, α, P, R* and R_b (the negative of the expected rate of inflation). The assumption that is commonly made--and the one made here--is that assets are gross substitutes both for households and for the economy as a whole. The model of bank behavior assures gross substitutability for the banks themselves.

Compensated Increases in Government Direct Loans

In the above model, an expansionary policy is one that causes q, the demand price of capital, to increase. From (A16) an expansionary policy can also be interpreted as one causing R_k to decrease. We now derive the condition for a compensated increase in government direct loans to be expansionary. Unfortunately, solving for comparative static implications even a modest system such as this is, in general, difficult and often counterproductive. Yet, under certain additional assumptions, the problems can be simplified considerably and certain qualitative results brought into focus. In particular, suppose that θ( ) takes the form

$$\theta(PL^*, GGL^*) = \theta(PL^*+GGL^*).$$

This will mean that, in equilibrium,[4/]

(A32) $R_{gg} = R_\ell + T + \Psi(\alpha).$

Furthermore, banks have a supply of total credit, rather than a supply of private or guaranteed loans per se. This supply takes the form

(A13e') $L^* = PL^* + GGL^* = b_\ell(R_g, R_b, R_d, \max(R_\ell, R_{gg}+T+\Psi(\alpha))D^*.$

Making use of (A12), (A29') and competition (so that (A32) holds), (A22) becomes

(A22') $-a_\ell(R^h)V + b_\ell(R_g, R_b, R_\ell, T+\Psi(\alpha))D^* = -GL^*$

and, because $b_{gg}(\ )$ is no longer defined, (A24) is replaced by

(A24") $b_\ell(R_g, R_b, R_\ell, T+\Psi(\alpha))D^* - GGL^* = PL^*$.

The 17 equation system, (A12), (A15)-(A21), (A22'), (A23), (A24'), (A24"), (A25)-(A29) can, upon substitution, be reduced to a system of four equations,

(A33) $a_k(R^{h'})[R^*K/R_k+\underline{B}^*+\underline{G}^*_n] = R^*K/R_k$

(A34) $[a_g(R^{h'})+b_g(R^{b'})a_d(R^{h'})][R^*K/R_k+\underline{B}^*+\underline{G}^*_n] = \underline{G}^*_n + \underline{GL}^*$

(A35) $[a_b(R^{h'})+b_b(R^{b'})a_d(R^{h'})][R^*K/R_k+\underline{B}^*+\underline{G}^*_n] = \underline{B}^*$

(A36) $[-a_\ell(R^{h'})+b_\ell(R^{b'})a_d(R^{h'})][R^*K/R_k+\underline{B}^*+\underline{G}^*_n] = -\underline{GL}^*$

in the three unknowns, R_k, R_g and R_ℓ, where

$R^{h'} = (R_k, R_g, R_b, R_\ell, (\underline{T}+\Psi(\alpha))(\underline{GGL}^*+\underline{GL}^*))$

and $R^{b'} = (R_g, R_b, R_\ell, T+\Psi(\alpha))$. Only three of these equations are independent so use (A33), (A34) and (A35) to conduct the comparative static exercises. To do so, differentiate these with respect to R_k, R_g, R_ℓ, $\underline{GL}^*$ and $\underline{GGL}^*$. The resultant system can be represented in matrix form as:

(A37)
$$H \begin{matrix} dR_k \\ dR_g \\ dR_\ell \end{matrix} \equiv \begin{matrix} h_{11}, & h_{12}, & h_{13} \\ h_{21}, & h_{22}, & h_{23} \\ h_{31}, & h_{32}, & h_{33} \end{matrix} \quad \begin{matrix} dR_k \\ dR_g \\ dR_\ell \end{matrix} = \begin{matrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ A_{31} & A_{32} \end{matrix} \quad \begin{matrix} d\underline{GGL}^* \\ d\underline{GL}^* \end{matrix} \ A \ \begin{matrix} d\underline{GGL}^* \\ Ad\underline{GL}^* \end{matrix}$$

Using the convention $c_{xy} = \partial c_x/\partial R_y$, the elements of H and their hypothesized signs (from gross substitutability) are:

$$\text{(A38)}\quad \begin{aligned} h_{11} &= a_{kk}V + (1-a_k)qK/R_k > 0 \\ h_{12} &= a_{kg}V < 0 \\ h_{13} &= a_{k\ell}V < 0 \\ h_{21} &= (a_{gk}+b_g a_{dk})V - (a_g+b_g a_d)qK/R_k < 0 \\ h_{22} &= (a_{gg}+b_{gg}a_d+b_g a_{dg})V > 0 \\ h_{23} &= (a_{g\ell}+b_{g\ell}a_d+b_g a_{d\ell})V < 0 \\ h_{31} &= (a_{bk}+b_b a_{dk})V - (a_b+b_b a_d)qK/R_k < 0 \\ h_{32} &= (a_{bg}+b_{bg}a_d+b_b a_{dg})V < 0 \\ h_{33} &= (a_{b\ell}+b_{b\ell}a_d+b_b a_{d\ell})V < 0. \end{aligned}$$

The elements of A, and their signs, are:

$$\text{(A39)}\quad \begin{aligned} A_{11} &= -(\underline{T}+\psi(\alpha))a_{k\gamma}V < 0 \\ A_{12} &= -(\underline{T}+\psi(\alpha))a_{k\gamma}V < 0 \\ A_{21} &= -(\underline{T}+\psi(\alpha))a_{g\gamma}V \leq 0 \\ A_{22} &= 1-(\underline{T}+\psi(\alpha))a_{g\gamma}V > 0 \\ A_{31} &= -(\underline{T}+\psi(\alpha))a_{b\gamma}V \leq 0 \\ A_{32} &= -(\underline{T}+\psi(\alpha))a_{b\gamma}V \leq 0. \end{aligned}$$

A compensated increase in government direct loans requires $d\underline{GL}^* = -d\underline{GGL}^* > 0$. Therefore,

$$\text{(A40)}\quad dR_k/dGL\,|_{dGGL=-dGL} = \{a_{k\ell}(a_{bg}+a_d b_{dg}+b_b a_{dg}) - a_{kg}(a_{b\ell}+b_{b\ell}a_d+b_b a_{d\ell})\}V/|H|.$$

Under the assumption that assets are gross substitutes all the partial derivatives in the expression in braces are negative, as is $|H|$.

Thus, the condition for the compensated increase in government direct loans to be expansionary is that

(A41) $\{a_{k\ell}(a_{bg}+a_d b_{bg}+b_b a_{dg}) - a_{kg}(a_{b\ell}+a_d b_{b\ell}+b_b a_{d\ell})\} > 0.$

In words, this means that sufficient conditions for an increase in government direct loans to be expansionary is that the demand for capital is more responsive to loan rates than to government security yields, and that the demands for the monetary base and deposits be more responsive to government security rates than to loan rates.

Increases in the Government Guarantee and Direct Loan Programs

The system (A37)-(A39) can also be used to address the question of the effects of uncompensated increases in the direct loan and guarantee programs on R_g, R_ℓ, and q. Unfortunately, without some additional assumptions, the resulting expressions defy simple interpretations. To make some headway, therefore assume that any increase in γ is only used to increase the demand for capital.[5/] That is:

(A42) $a_{g\gamma} = a_{b\gamma} = 0,\ a_{k\gamma} > 0.$

Questions of interest are whether increases in GGL* and GL* displace private borrowing of at least some agents (i.e., $dR_\ell/dGGL^*$, $dR_\ell/dGL^* > 0$) and the effects on q.

First, consider $dR_\ell/dGGL^*$. Using (A37)-(A39) and (A42) gives

(A43) $R_\ell/dGGL^* = A_{11}\ [h_{21}h_{32} - h_{31}h_{22}]/|H|$

which is positive since, from (A38), the term in brackets is positive and A_{11} and $|H|$ are negative. Thus, some private borrowing by non-insured borrowers is displaced by increases in the government guarantee programs. This result need not be the case if the direct loan program is expanded instead. In that case:

(A44) $dR_\ell/dGL^* = dR_\ell/dGGL^* + A_{22}[h_{12}h_{31} - h_{11}h_{32}]/|H|.$

The numerator of the second term is positive and the denominator is negative. Thus, $dR_\ell/dGL^* < dR_\ell/dGGL^*$ and it is also possible for (A44) to be negative.

Finally, $dR_k/dGGL^*$ is given by the expression

(A45) $dR_k/dGGL^* = A_{11}[h_{22}h_{33} - h_{23}h_{31}]/|H| < 0.$

Therefore, increases in the government guarantee program increase the demand price of capital. From (A40) and (A45), so too will increases in the direct loan program if the condition (A41) is met.[6]

FOOTNOTES

[1]The portfolio choice model developed here is based upon that developed by James Tobin, "A General Equilibrium Approach to Monetary Theory," Journal of Money, Credit and Banking (February 1969), pp. 15-29.

[2]See Joel Fried and Peter Howitt, "The Effects of Inflation and Real Interest Rates," American Economic Review, forthcoming.

[3]R_d also depends upon Δ_ℓ amd Δ_g. These, however, will be assumed constant throughout the analysis and will therefore be ignored.

[4]Note that under this assumption, the setting of T or of S is equivalent with $\underline{S} = \underline{T} + \Psi(\alpha)$.

[5]Note that this assumption is not necessary to examine compensated changes in GL given the form of γ.

[6]Examining the subsequent adjustments when prices are free to move is beyond the scope of this appendix, although certain general principles can be sketched. In a one sector model, perfectly flexible prices would imply that the demand price for capital would return to its initial level. In a two sector model, this need not be the case, but the results will depend upon distribution effects. If the individuals receiving the greater purchasing power--e.g., the non-insured borrower in the compensated increase in direct loans case--have a greater propensity to purchase capital than the

population at large, then it would be expected that the demand price of capital will rise (as will R_g) in the new equilibrium.