ECONOMIC RESEARCH
FEDERAL RESERVE BANK OF ST. LOUIS

WORKING PAPER SERIES

The Influence of Fiscal and Monetary Actions On Aggregate Demand: A Quantitative Appraisal

Authors	Keith M. Carlson, and Denis S. Karnosky
Working Paper Number	1968-004A
Creation Date	January 1968
Citable Link	https://doi.org/10.20955/wp.1968.004
Suggested Citation	Carlson, K.M., Karnosky, D.S., 1968; The Influence of Fiscal and Monetary Actions On Aggregate Demand: A Quantitative Appraisal, Federal Reserve Bank of St. Louis Working Paper 1968-004. URL https://doi.org/10.20955/wp.1968.004

Federal Reserve Bank of St. Louis, Research Division, P.O. Box 442, St. Louis, MO 63166

The views expressed in this paper are those of the author(s) and do not necessarily reflect the views of the Federal Reserve System, the Board of Governors, or the regional Federal Reserve Banks. Federal Reserve Bank of St. Louis Working Papers are preliminary materials circulated to stimulate discussion and critical comment.

Working Paper No. 4

The Influence of Fiscal and Monetary Actions On Aggregate Demand: A Quantitative Appraisal

Keith M. Carlson
Denis S. Karnosky
March, 1969

Paper Originally Prepared for the Fall Meeting of the Federal Reserve System Committee on Financial Analysis, Federal Reserve Bank of San Francisco, October 26, 1967.

This Working Paper is circulated for discussion and comment. Quotations and references only by permission of the authors.

CONTENTS

THE INFLUENCE OF FISCAL AND MONETARY ACTIONS ON AGGREGATE DEMAND: A QUANTITATIVE APPRAISAL *

Economic developments since 1966 have produced renewed controversy about the relative influence of monetary and fiscal actions on the level of economic activity. The recent continued advance of GNP in the face of ostensibly restrictive Federal budget actions has raised questions about the effectiveness of fiscal action as a stabilization tool. 1/ Meanwhile, monetary policy has come to the forefront as a potentially powerful influence on economic activity.

---

* The original version of this paper has been revised to take some account of research efforts since late 1967. Results published since then have provided added support to the monetarist interpretation of economic events. The results given here tend to substantiate such an interpretation for the short run, but for longer periods the conclusions about the relative influence of fiscal and monetary actions are less clear.

1/ Even before the 1968 tax increase, there were papers that questioned fiscal policy en route to championing the cause of monetary policy. Examples are Allan Meltzer, "Money Managers and the Boom," Challenge (March-April, 1966), pp. 4-7; Yale Brozen, "The Mythology of the New Economics," The Banker (August, 1967), pp. 678-83; James Meigs, "A New Look at Monetary and Fiscal Policy -- Three Views," unpublished paper presented before National Association of Business Economists in Detroit, Michigan, September 29, 1967; David Meiselman, "The New Economics and Monetary Policy," unpublished paper presented before the Management Conference of the Graduate School of Business of the University of Chicago, Illinois, March 8, 1967; John M. Culbertson, statement prepared for the Joint Economic Committee Hearings on the 1967 Economic Report of the President, February 16, 1967. A more recent example is L.C. Andersen and J.L. Jordan, "Monetary and Fiscal Actions: A Test of Their Relative Importance in Economic Stabilization," Federal Reserve Bank of St. Louis Review (November, 1968). It might be noted, however, that the recent discussion is a variation of a theme that raged in the literature in 1963 and 1964, i.e., the Friedman-Meiselman-Ando-Modigliani-Hester-Deprano-Mayer controversy.

Critics of fiscal policy have noted the low degree of significance of the fiscal variables when used in multiple regression analysis, and conclude that budget policy is not predictably related to GNP. Similarly, a higher degree of significance between changes in the money supply (or some other monetary magnitude) and changes in GNP is used as support for the effectiveness of monetary policy. Such conclusions may be correct, but they do not logically follow from the empirical analysis that precedes them.

A standard criticism of the approach of regressing GNP directly on monetary and fiscal variables is that a truly effective policy (monetary or fiscal) would necessarily be unrelated to the observed data on GNP. [2/] If the policy variable were changed to offset all movements (e.g., random shocks) tending to divert GNP from its high-employment growth path, a low degree of significance between the policy variable and GNP would be expected.

A proper analysis of monetary and fiscal policy would seem to require some indication of economic structure. To assess the impact of monetary and fiscal variables on some economic variable, say GNP, one needs to specify the other factors influencing GNP before the impact of monetary and fiscal actions can be isolated. It is in this spirit that a structural economic model is presented. More specifically, this paper is addressed to the following questions:

2/ Such an example is demonstrated in John Kareken and Robert Solow, "Part I. Lags in Monetary Policy," in Stabilization Policies, a series of Research Studies Prepared for the Commission on Money and Credit (Englewood Ciffs: Prentice-Hall, 1963), p. 16.

1. What is the impact of budget policy on the economy?
2. How does this impact vary with changes in certain key behavioral parameters in the economic system?
3. How does the impact vary under different patterns of financing the budget position?

Monetary policy enters the discussion because fiscal policy has monetary effects. A point of emphasis here is that monetary and fiscal policy are not independent and should always be viewed jointly. 3/

The objective of this paper is to provide quantitative answers to the questions posed above. The existing literature is surveyed briefly to provide some indication of the current state of knowledge on the economic impact of fiscal and monetary actions. A sensitivity analysis is conducted to determine the responsiveness of certain reduced-form multipliers to changes in the behavioral parameters of an economic model. The model used in this analysis is motivated by the work of Carl Christ, which, as an aside, is modified to examine monetary-fiscal mix in the context of GNP and interest rate targets. 4/ A model is then estimated for 1952-67. The interpretation of these

---

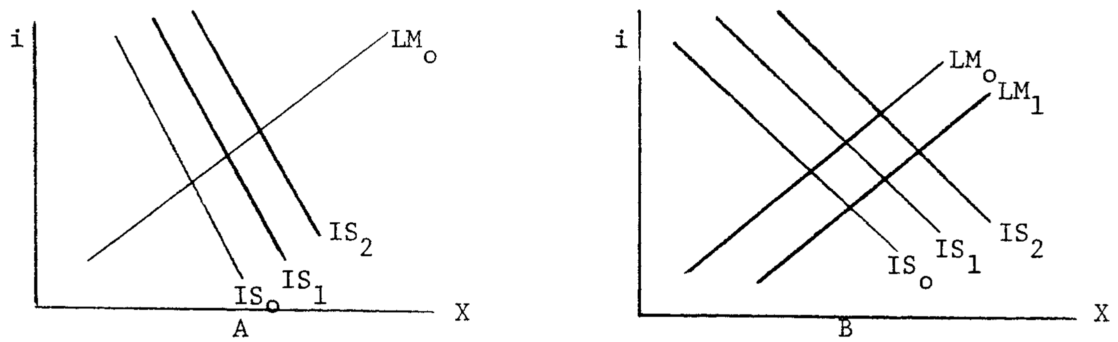
3/ This point is well known and is generally made clear in most macro-economic textbooks. Detailed discussion can be found in Richard Musgrave, The Theory of Public Finance (New York: McGraw-Hill, 1958), Chapter 22. This interdependence seems to have been forgotten at times, however. An attempt to restore this emphasis is found in Carl Christ, "A Short-Run Aggregate Demand Model of the Interdependence and Effects of Monetary and Fiscal Policies with Keynesian and Classical Interest Elasticities," American Economic Review, Papers and Proceedings (May, 1967), pp. 434-443, and "A Simple Macroeconomic Model with a Government Budget Restraint," Journal of Political Economy (January/February, 1968), pp. 53-67.

4/ Ibid.

results focuses on the varying impact of alternative methods of financing government expenditures.

To clarify the objectives of the paper, an illustration is offered.

Consider the conventional Hicksian IS - LM diagram.



Suppose the government increases its purchases of goods and services, shifting the IS curve to the right (Panel A). Conventional analysis stops here, concluding that increased government spending leads to higher levels of NNP and interest rates. Ignored is the question of how the government obtained its funds. If it is assumed that debt is sold, and the proceeds are spent, the money stock, and therefore LM, is unchanged. However, the public then holds additional debt, which will probably have effects on private spending (shifting IS from IS_1 to IS_2). The result of the fiscal action is an increase in both income and the interest rate. If, however, the central bank undertakes to finance the increased expenditures by creating high-powered money, the IS curve will shift as before, but, in addition, the LM curve will shift (from LM_o to LM_1 in Panel B). As a result there is a larger increase in income than in the previous case, and the effect on the interest rate is ambiguous. 5/ The analysis could

5/ Clearly the illustration here is one of short-run equilibrium. Long-run equilibrium requires a balance in the government's budget, because income tends to change via the asset effect as long as assets are changing, which is the case if the government is running a surplus or deficit. For a discussion of these conditions, see David J. Ott and Attiat Ott, "Budget Balance and Equilibrium Income," Journal of Finance (March, 1965), pp. 71-77.

be carried further by considering the effects of increased NNP on tax revenues, debt retirement, and changes in asset holdings of the public. The point to be made is that monetary and expenditure effects are operating simultaneously, and the new equilibrium levels of interest rate and NNP depend on the relative shifts of the schedules.

The purpose of this paper is to provide some indication of the relative strength of these different effects. Since these effects are operating simultaneously, an attempt to measure their strength requires the specification of an aggregate economic model in which a set of endogenous variables are simultaneously determined.

I. A Cursory Review of the Literature

Generally, aggregate economic models do not explicitly specify government financing considerations. Expenditures are considered as to their effect on other variables, but the acquisition of funds, other than induced revenue changes, is not usually considered. Such an omission indicates that such models fail to account fully for the influence of fiscal policy on aggregate demand, interest rates, etc. Alternative methods of financing expenditures have different effects, thus any analysis of fiscal impact logically should consider the means of financing.

A. Modigliani and Monetary-Fiscal Mix

Modigliani has presented a theoretical model which includes a government budget identity. [6/] Through this identity, Modigliani

---

6/ Franco Modigliani, "The Monetary Mechanism and Its Interaction with Real Phenomena," Review of Economics and Statistics, Supplement (February, 1963), pp. 79-107.

emphasizes that "monetary and fiscal policies can be made entirely independent of each other." 7/ This statement is correct, however, because he defines monetary and fiscal policies to exclude debt management (which he defines as changing the amount of government debt held by the public). Modigliani's interest in monetary-fiscal policy mix rests on the consideration of deficit financing (implying a change in the amount of debt outstanding) and its implications as a burden on future generations. Increased government borrowing tends to be accompanied by higher interest rates and reduced investment, thus a smaller capital stock is passed on to future generations than would be otherwise. As a result, Modigliani concludes "that the case for a currently balanced budget, and hence for relying on monetary rather than on fiscal policy as a first line of defense in controlling aggregate demand, is somewhat stronger than might have appeared some time ago." 8/

The qualitative analysis of the Modigliani paper serves as motivation for this study - a specific question of interest being the quantitative effect of different budget financing patterns on investment.

B. Macroeconometric Studies

The professional literature abounds with macroeconometric studies based on theoretical models similar to Modigliani's (except

---

7/ Ibid, p. 94.

8/ Ibid., p. 97. See also Modigliani's article, "Long-Run Implications of Alternative Fiscal Policies and the Burden of the National Debt," Economic Journal (December, 1961), pp. 411-433.

for consideration of the budget identity). Most of these estimated models verify the importance of both monetary and fiscal policy in the determination of economic activity.

A very short and selected list of macroeconometric studies is summarized in Table I. Comparisons are made difficult, however, by substantial differences in the specification of the underlying models.

Estimates of the multipliers show considerable uniformity once allowance is made for the different characteristics of the models, time periods, data, etc. If the policymaker were to use these models as a guide in the formation of policy, there would seem to be little difficulty in terms of choosing direction. Problems would probably arise in the determination of the magnitude of policy action, however, even aside from defining the policy variable in a particular way. All of the above models fail to discuss explicitly the government's financing problem. 9/

C. Shortcomings of Macroeconometric Studies

This survey is certainly incomplete and probably unfair. Nevertheless, it is meant to demonstrate some major shortcomings of conventional macroeconometric efforts. One is that financing of government

---

9/ The failure to emphasize financing considerations is not meant to be a criticism of the structure of the models. Financing enters the discussion when the models are manipulated to determine the effects of changes in policy variables. It is this lack of emphasis in the manipulation of the model that provides the basis for criticism here. This point was clarified by Lyle Gramley.

TABLE I

Summary of Selected Macroeconometric Models

	Scott	Chow	Hanrahan	Ando-Goldfeld
Data	Quarterly	Annual	Quarterly	Quarterly
Period	1951-64	1930-40 1948-63	1952-60	1950-64
Multiplier	Quarterly Impact	Annual Impact	Quarterly Impact	Annual Total
Income Variable	GNP (current $'s)	GNP (current $'s)	GNP (1958 $'s)	GNP (current $'s)
Int. Rate Variable	Corp. Aaa	10 Yr.Corp.Bonds	Weighted Avg. of Short & Long Gov't.	a) 90-Day Bills b) 10-Yr.Gov't.
Change in Income/ Change in Gov't. Spending	1.20	1.62	2.24	2.33
Gov't. Spending Variable	Gov't.Spending & Net Exports	Purchases of goods & Services	Real Expend. at all Levels of Gov't.	Expenditure at all Levels & Net For. Inv.
Change in Int.Rate/ Change in Gov't. Spending	.027	.025	.073	a) .279 b) .130
Change in Income/ Change in Monetary Variable	1.90	1.72	1.49	2.66
Monetary Variable	Currency and Time & Demand Deposits	Currency and Demand Deposits	Nonborrowed Reserves	Federal Reserve Credit
Change in Int. Rate/ Change in Monetary Variable	- .001	- .054	- .084	a) -.119 b) -.151

Sources:

R. H. Scott, "Estimates of Hicksian IS and LM Curves for the United States," Journal of Finance (September, 1966), pp. 479-87.

Gregory Chow, "Multiplier, Accelerator, and Liquidity Preference in the Determination of National Income in the United States," Review of Economics and Statistics, (February, 1967), pp. 1-15.

G. D. Hanrahan, "Three Econometric Models of the U.S.," unpublished doctoral dissertation, University of Minnesota (August, 1964).

A. Ando and S. Goldfeld, "An Econometric Model for Evaluating Stabilization Policies," A. Ando, E.C. Brown and Ann F. Friedlander (editors), Studies in Economic Stabilization (Washington, D.C.: The Brookings Institution, 1968), pp. 215-87.

expenditures is almost always given short shrift in such models, yet some of the modern critics maintain that financing is a key factor underlying the determination of the impact of the budget on the economy. 10/

Another shortcoming concerns the usefulness of these models in the formulation of short-run stabilization policy. All calculated multipliers derived from such econometric studies are point estimates, indicating an average relationship for the time period under consideration. Furthermore, the impact of a particular policy action is analyzed for either a very short period or a very long one, shedding little light on the time path of the effect of a policy action.

A related criticism is the degree to which these multipliers are sensitive to behavioral parameters in the economic system. If, for example, the government-spending multiplier is sensitive to the elasticity of money demand with respect to income, the reliability of the multiplier is contingent on the reliability of the estimate of that elasticity. The following section examines the sensitivity of the multiplier and the quantitative interdependence of monetary and fiscal policy.

II. Interdependence and Effects of Fiscal and Monetary Actions: Some Implications of the Christ Model

The Christ model is a noteworthy attempt to consider the monetary effects of fiscal actions, as well as the conventional expenditure effects. Questionable features of the model may be noted, but

---

10/ Christ, *op. cit.*

it still serves as a useful point of departure in analyzing the full effects of fiscal actions. 11/

A. Summary of Christ Model

The model consists of eleven equations and is expressed in first differences. There are two sectors - government (including the treasury and the central bank) and private (including commercial banks and state and local governments). Three assets are included - physical capital, government bonds and high-powered money. The relation between sectors and assets is summarized by the following balance sheets:

Government Sector

Assets	Liabilities
	Government bonds held by private sector
	High-powered money

Commercial Banking Sector

Assets	Liabilities
High-powered money	Demand deposits
Government bonds	
Loans	

11/ See the discussion following the Christ, op. cit. The Christ model is summarized in considerable detail to make this paper as self-contained as possible. This summary is in no way meant to be a substitute for the original.

Private Sector

(Excluding Commercial Banks)

Assets	Liabilities and Net Worth
Demand deposits	Loans
High-powered money	Wealth
Government bonds	
Physical capital	

When banking sector is consolidated with the nonbank private sector, the balance sheet of the private sector as a whole is derived.

Private Sector

Assets	Liabilities and Net Worth
High-powered money	Wealth
Government bonds	
Physical capital	

The equations of the model are as follows:

(1) Real NNP definition:

$$\Delta x = \Delta c + \Delta i + \Delta g$$

where x: real net national product;

c: real consumption;

i: real net investment;

g: real government purchases.

(2) Real physical capital identity: 12/

$$\Delta k = \Delta i$$

12/ This identity follows from Christ's procedure of setting the lagged first differences of all variables equal to zero, which he claims has a negligible effect on his results. Noting the identity in terms of levels, $k = k_{-1} + i$; taking the first difference yields $k - k_{-1} = k_{-1} - k_{-2} + i - i_{-1} = \Delta k = \Delta k_{-1} + \Delta i$. Setting $\Delta k_{-1} = o$ gives equation (2).

where k: real physical capital.

(3) Real private wealth definition: 13/

$$\Delta w = \Delta k + \Delta H/P + \Delta B/rP - (B/r^2P)\ \Delta r$$

where w: real private wealth;

H: high-powered money stock;

P: price level;

B: number of government bonds held by private sector;

(4) Tax transfer equation:

$$\Delta t = \Delta t_o + t_1 \Delta x$$

where t: real tax receipts less transfers;

t_o: a tonomous real taxes less transfers;

t_1: marginal tax-transfer rate.

(5) Real disposable income definition

$$\Delta y = \Delta x - \Delta t$$

where y: real disposable income.

(6) Real capital gain on bonds: 14/

$$\Delta z = -\ (B/r^2P)\ \Delta r$$

where z: real capital gain on bonds;

r: yield on perpetual government bonds.

(7) Real consumption:

$$\Delta c = c_1 \Delta y + c_2 \Delta z + c_3 \Delta r + c_4 \Delta r'$$

13/ This follows from the definition in terms of levels: $w = k + H/P + B/rP$. Form the total differential, noting that P is a constant, $dw = dk + dH/P + (1/rP)\ dB - (B/rP)\ dr$, and write in discrete form.

14/ This follows from first differencing the definition of capital gains, $z = H\Delta\ (1/P) + B\Delta\ (1/rP)$.

where r': yield on physical capital.

(8) Real net investment:

$$\Delta i = i_1\Delta x + i_2\Delta r + i_3\Delta r'.$$

(9) Real high-powered money demanded:

$$(1/P)\Delta H = h_1\Delta x + h_2\Delta w + h_3\Delta r + h_4\Delta r'.$$

(10) Real government bonds demanded:

$$(1/rP)\Delta B = b_1\Delta x + b_2\Delta w + (b_3 + B/r^2P)\Delta r + b_4\Delta r'.$$

(11) Government budget identity:

$$\Delta g = \Delta t_o + t_1\Delta x + (1/P)\Delta H + (1/rP)\Delta B.$$

The endogenous variables of the model are Δc, Δi, Δk, Δr, $\Delta r'$, Δt, Δw, Δx, Δy, Δz and one of the four policy variables -- Δg, ΔH, Δt_o, ΔB. Whichever one of the policy variables is chosen as endogenous, the remaining three are exogenous, i.e., subject to the control of policymakers. In addition, P and t_1 are considered predetermined and fixed (i.e., exogenous).

This model differs from most macroeconomic models in several respects. First, portfolio selection is explicitly considered and includes three assets. Unlike most models, real physical capital is included in the portfolio, and behaves as a shadow market in the model, i.e., a demand function for physical capital is not explicitly specified. This is an extension of simple two-asset Keynesian models where demand functions for bonds are not included, yet the yield on bonds is determined by the model. Such an equation is redundant by Walras' Law.

Another difference is the specification of a government budget identity. Equation (11) says that government purchases must be financed by some combination of taxes, debt issue or high-powered money issue. This identity serves as a constraint allowing the policymakers n-1 degrees of freedom in the setting of n monetary and fiscal variables. A policy is then defined as a set of assigned values for n-1 policy variables.

From this constraint, the multiplier effect of a given change in a particular policy variable depends on the values assigned to the remaining n-2 policy variables. For example, Christ shows that the multiplier effect of increased government purchases on aggregate demand can vary from 1.11 under pure tax finance (induced taxes are counted as a source of revenue) to 6.16 under pure high-powered money finance (autonomous taxes are decreased to counteract the induced portion of tax revenue).

B. Christ's Results

Equation (11) indicates that the model contains four policy variables, three of which can be allowed to vary independently. Assuming values for the parameters of the model, Christ derives reduced form equations (i.e., solves for the endogenous variables in terms of the exogenous variables) for a number of different cases. Consider first the case where ΔB, change in bond holdings, is treated as an endogenous variable. The reduced form multipliers are shown in Table II.

Table II

ΔB Endogenous

	Δg	Δt_o	ΔH
Δx	2.42	-1.69	1.66
Δk	.26	- .17	.12
Δw	.31	- .37	1.41
Δt	.48	.66	.33
Δy	1.94	-2.35	1.33
Δz	- .47	.46	1.63
Δc	1.16	-1.51	1.54
Δi	.26	- .17	.12
Δr	.006	- .006	- .023
$\Delta r'$	.037	.017	- .109
ΔB	.51	- .66	-1.33

Each of the values in Table II is a reduced form multiplier, e.g., a unit change in g has a multiplier effect on the endogenous variable specified. All variables, except the yields Δr and $\Delta r'$, are measured in dollars. The yields are in percentages, thus the value .006, associating Δr with Δg, means $1 billion of Δg has the effect of raising r by six-tenths of a basis point.

It should also be noted that Christ uses a comparative statics model, thus his results represent changes in equilibrium values, with no consideration given to the path of change to a new equilibrium. Although Christ does not make it explicit, it seems that most of his assumed parameters are taken from studies using annual data, so the multipliers might be considered simultaneously as annual impact and steady state multipliers.

The case summarized in Table II is presented first and discussed at some length because it seems to approximate the institutional framework of the United States. Fiscal policy is normally thought of in terms of taxing and spending, and monetary policy in terms of altering the stock of high-powered money (or some closely related magnitude). [15/] The amount of debt held by the private sector is not commonly viewed as a stabilization policy variable, although it certainly can be.

Consider the first column of Table II. A change of $1 billion in government purchases of goods and services, with no tax action or high-powered money issue, produced $.48 billion in tax revenue. The remainder, $.51 billion, must be financed by debt issue. These results suggest that, *ceteris paribus*, about half of government spending is "self-financed."

The tax column (second column in Table II) is similar to the government spending column with a couple of exceptions. For conventional reasons the multipliers are generally smaller in absolute value. A $1 billion *shift* upward of the tax function lowers x, i.e., real NNP, by $1.69 billion, but in fact produces only $.66 billion in additional

---

15/ See Leonall Andersen, "Three Approaches to Money Stock Determination," Federal Reserve Bank of St. Louis *Review* (October, 1967), pp. 6-13.

tax revenue. Consequently, if government purchases (g) and high-powered money (H) are kept constant, privately-held government debt must change by the same amount ($.66 billion), but in the opposite direction. The effect of the government retiring some of its debt is negligible, lowering interest rates (r) by 0.6 of a basis point.

When spending and taxing policies are viewed jointly, it is noted that there is a "balanced budget" multiplier effect on x of .73 (2.42 - 1.69), but this calculation makes no allowance for induced tax revenues. If g and t_o are changed to produce a balance (assuming the budget is initially balanced), t_o needs to be changed only $.79 billion for each $1 billion of g. Following this policy, x is changed by 1.11 [2.42 - .78 (1.69)]. Interest rates are essentially unaffected under the latter policy, rising 1 basis point.

Finally, the implications of these results for monetary policy can be examined by noting the last column of Table II. The effect of a $1 billion change in high-powered money, H, is to increase x by $1.66 billion. This result is obtained in the following way: to increase H, the central bank purchases bonds in the open market. Interest rates must fall to induce portfolio adjustments; lower interest rates induce increased consumption and investment, and thus income and tax revenues also increase; the induced revenues are used to retire debt, placing additional downward pressure on interest rates. To persuade the private sector to hold fewer bonds at higher income levels, the interest rate must fall by 2.3 basis points to reach the new equilibrium. Interestingly, consumption is more responsive to such monetary action than is investment. The explanation lies in the specification of the consumption and investment functions, equations (7) and (8). Since the multiplier is quite large for capital gains, z, consumption is affected in that way, and investment is not. Furthermore, Christ assumed $c_3 > i_2$, thus the interest rate decline encourages consumption more than investment. Also, r' enters (7) negatively and (8) positively, thus a decline in the yield on real capital tends to encourage consumption, but discourage investment. Table II raises some interesting questions about the mix of monetary and fiscal policy. Such a discussion is deferred momentarily so as to present the remainder of the cases derived by Christ.

The remaining cases (ΔH, Δt_o and Δg endogenous) are probably of only academic interest currently, but may be representative of past periods in U.S. history. For example, debt considerations (although legal debt limits do not usually refer to debt in the hands of only the private sector) may be overriding, requiring passive action with respect to g, t_o or H.

The results of the other cases are presented in Tables III-V without further comment.

Table III

ΔH Endogenous

	Δg	Δt_o	ΔB
Δx	3.08	-2.51	-1.24
Δk	.31	- .23	- .09
Δw	.86	-1.08	-1.06
Δt	.62	.50	- .25
Δy	2.45	-3.01	-1.00
Δz	.16	- .35	-1.22
Δc	1.76	-2.28	-1.16
Δi	.31	- .23	- .09
Δr	- .002	.005	.017
$\Delta r'$	- .005	.037	.082
ΔH	.39	- .50	- .75

Table IV

Δt_o Endogenous

	Δg	ΔH	ΔB
Δx	1.11	5.05	2.55
Δk	.13	.46	.26
Δw	.02	2.16	.56
Δt	1.00	-1.00	-1.00
Δy	.11	6.05	3.55
Δz	- .11	.70	- .70
Δc	- .01	4.58	2.29
Δi	.13	.46	.26
Δr	.001	- .010	.010
$\Delta r'$	.024	- .074	.026
Δt_o	.78	-2.01	-1.51

Table V

Δg Endogenous

	Δt_o	ΔH	ΔB
Δx	1.43	7.92	4.70
Δk	.16	.79	.51
Δw	.03	2.21	.60
Δt	1.29	1.58	.94
Δy	.14	6.34	3.76
Δz	- .14	.42	- .90
Δc	- .02	4.54	2.26
Δi	.16	.79	.51
Δr	.002	- .006	.012
$\Delta r'$	.030	- .013	.072
ΔH	1.29	2.58	1.94

C. Monetary-Fiscal Mix and an NNP Target

The Christ model can be extended to deal with the mix of monetary-fiscal policy required to achieve a given target. If the change in real NNP is selected as a target variable, policymakers lose one degree of freedom in the selection of values for policy variables. In this model, with Δx given, only two of the four policy variables are subject to control if equilibrium is to be achieved. The case where Δg and ΔB are the predetermined policy variables (and Δt_o and ΔH are endogenous) is summarized in Table VI.

Table VI

Δx Target Variable; ΔH and Δt_o Endogenous

	Δg	ΔB	Δx
Δk	.02	.06	.09
Δw	- .46	- .02	.43
Δt	1.22	- .50	- .20
Δy	-1.22	.50	1.20
Δz	- .26	- .58	.14
Δc	-1.02	- .06	.91
Δi	.024	.06	.09
Δr	.004	.008	- .002
$\Delta r'$	.053	.132	- .020
ΔH	- .22	- .50	.20
Δt_o	1.22	- .50	- .40

The implications of this model for monetary-fiscal policy mix are summarized in terms of a numerical illustration. It is assumed that the desired Δx is \$50 billion and Δg is assumed equal to \$10 billion. The exact change in H and t_o required to achieve equilibrium depends on the value chosen for ΔB. Some alternatives are summarized in Table VII. The effects on c, i and r are also noted.

Table VII

Δx=50; Δg=10; ΔH and Δt_o Endogenous

ΔB	ΔH	Δt_o	Δc	Δi	Δr
-10	12.8	- 2.8	35.8	4.2	-.139
- 5	10.3	- 5.3	35.6	4.4	-.099
0	7.8	- 7.8	35.3	4.7	-.059
5	5.3	-10.3	35.0	5.0	-.019
10	2.8	-12.8	34.8	5.2	+.021
15.6	0.0	-15.6	34.3	5.7	+.066
16.4	- 0.4	-16.0	34.2	5.8	+.072

In this case the mix of tax and monetary policy (i.e., controlling the stock of high-powered money) is outlined (given Δg = \$10 billion and a policy target of Δx = \$50 billion). An interesting feature of this case is that tax cuts are required even with large increases in H. (Remember that this case assumes that the budget is initially in balance.) The large variation in ΔB correspondingly induces large changes in interest rates, i.e., from ΔB = -\$10 billion to ΔB = \$10 billion; the equilibrium interest rate varies by 16 basis points. But unless you are interested in interest rates _per se_, the effect on consumption and investment is just the opposite of what might be expected. A relatively austere tax policy with easy money increases consumption more than the opposite mix. The reason again can be traced back to the specification of the consumption and investment functions, and in particular the relatively greater responsiveness of consumption to interest rates.

D. Monetary-Fiscal Mix and Two Targets

A final case considered is that of two targets, Δx and Δr. Such a situation reduces the independent policy variables to one. This case is summarized in Table VIII with Δg treated as the exogenous policy variable.

Table VIII

Δx and Δr Target Variables; ΔH, ΔB and Δt_o Endogenous

	Δg	Δx	Δr
Δk	- .00009	.10	- 1127.6
Δw	- .43	.41	-11080.6
Δt	1.43	- .31	2753.0
Δy	-1.43	1.31	- 2753.0
Δz	0.0	0.0	- 7200.0
Δc	-1.0	.90	1127.6
Δi	- .00009	.10	- 1127.6
$\Delta r'$	- .000001	.00007	- 13.1
ΔH	- .01	.09	2615.5
ΔB	- .42	.22	- 5368.4
Δt_o	1.42	- .51	2753.0

Again, to help make sense out of this case, a numerical illustration is developed. Target Δx is set equal to \$50 billion, Δr equal to zero, and implied values for ΔH, ΔB, and Δt_o are derived when Δg is allowed to vary.

Table IX

$\Delta x=50$; $\Delta r=0$; ΔH, ΔB and Δt_o Endogenous

Δg	ΔH	ΔB	Δt_o
0	4.50	11.0	-25.5
5	4.45	8.9	-18.4
10	4.40	6.8	-11.3

The results listed in Table IX are somewhat as expected, given the values in Table VII. The interesting feature of this case is that given the values of Δg, a substantial tax cut is required to enable ΔH and ΔB to be increased so as to satisfy the greater demand for H and B at the higher target level of income while maintaining an unchanged interest rate. Such considerations that arise from a simultaneous system with a government budget constraint seldom receive emphasis in policy discussions.

E. Sensitivity Analysis of Reduced Form Multipliers

The above discussion shows that reduced form multipliers assume a wide range of values under different institutional and policy situations. Since all of Christ's multipliers were calculated from assumed values of structural parameters (although based on other empirical studies), the question naturally arises as to the reliability of the parameters. In other words, it would be helpful to know the standard errors of the parameters. These could be found in two ways, (1) the studies from which the values were taken could be tracked down, or (2) the model could be estimated to derive new estimates with associated standard errors. Alternative (1) is subject to the shortcomings mentioned previously: independent studies vary in model specification and data definition. Alternative (2) does not provide the results of thorough and detailed studies of particular sectors (an advantage of alternative (1)), and all reduced form coefficients would require recalculation in light of any new estimates of structural parameters.

The procedure followed here parallels that of alternative (2) more closely than (1). To assist in such an analysis, it is of value to know which parameters are most critical in determining the magnitude of the reduced form multipliers. A large standard error associated with a critical parameter assumes great significance. It is not only important to identify the critical parameters, but it is also helpful to know the degree to which certain multipliers are sensitive to these parameters. Such information would seem to be needed to properly implement policy.

Christ devoted some emphasis to the interest elasticity of investment and its effect on the reduced form multipliers for real NNP. Varying this elasticity from 0 to -1, he concluded that real NNP multiplier effects of spending and tax policies are not very sensitive to this elasticity in the relevant range. The NNP multiplier effect of open market operations, however, was sensitive toward the Keynesian extremes where interest elasticities are low, i.e., approach zero.

The relation between interest elasticity of high-powered money demand and the real NNP multiplier was also examined by Christ. In this case, the elasticity was varied from -.2 to $-\infty$. As above, the real NNP multiplier effects of spending and tax policies were found to be relatively insensitive to variations in elasticity. Similarly, the NNP multiplier effect of open market operations was sensitive in the Keynesian range of high elasticity.

Turning now to a more inclusive sensitivity analysis of the reduced form multipliers, the critical parameters are brought into sharper focus. The sensitivity analysis was conducted by individually increasing each structural parameter in the Christ model and computing the resulting reduced form. This procedure was repeated for each of the four policy sets, i.e., where ΔB, ΔH, Δt_o and Δg were, in turn, considered endogenous. The computations yielded 64 sets of reduced form equations. The results were then compared with the respective original reduced form set, and percentage changes in the multipliers were computed. The results for the policy set with ΔH endogenous is recorded in Table X.

To assist in the interpretation of the table, the magnitudes of the reduced form multipliers are written in the left-hand column. The small size of some of these multipliers explains why many of the per cent changes vary so greatly.

Extracting the critical parameters from Table X enables the construction of a smaller table (Table XI). This table identifies the parameters of the model which would seem to be critical from the standpoint of policymakers.

TABLE X

Sensitivity Analysis of Reduced Form Multipliers [1]

(Per cent change in multiplier associated with 10 per cent change in parameter)

	Base Value	∂c/∂y	∂c/∂z	∂c/∂r	∂c/∂r'	∂i/∂x	∂i/∂r	∂i/∂r'	∂B/∂x	∂B/∂W	∂B/∂r	∂B/∂r'	∂H/∂x	∂H/∂w	∂H/∂r	∂t/∂x
d( )/dg																
x	3.06	11.52	.07	.09	.09	1.85	.02	-.02	-0-	-0-	.01	-0-	-1.24	-.11	-.17	-4.78
k	.31	15.70	.09	.12	.13	12.78	.14	-.16	.14	.10	-.28	.05	-.52	-.04	-.07	-4.40
w	.86	-16.36	-.10	-.07	-.14	1.11	.01	-.02	.73	.51	-1.40	.26	-1.95	-.17	-.26	-7.33
t	.61	11.52	.07	.09	.09	1.85	.02	-.02	-0-	-0-	.01	-0-	-1.24	-.11	-.17	4.74
y	2.45	11.52	.07	.09	.10	1.85	.02	-.02	-0-	-0-	.01	-0-	-1.24	-.11	-.16	7.16
z	.16	-71.85	-.43	-.54	-.59	-11.18	-.12	.14	3.53	2.48	-6.81	1.27	-13.87	-1.17	-1.84	-12.41
c	1.76	17.34	.10	.13	.14	.99	.01	-.01	-.03	-.02	.06	-.01	-2.07	-.18	-.28	-7.57
i	.31	15.70	.09	.12	.13	12.78	.14	-.16	.14	.10	-.28	.05	-.52	-.04	-.07	-4.40
r	-.000023	-71.86	-.44	-.57	-.62	-11.17	-.13	.13	3.52	2.46	-6.82	1.28	-13.85	-1.19	-1.85	-12.40
r'	-.000050	-301.79	(∓)[2] -2.14	-2.82	-3.08	-69.34	-.52	1.09	-8.39	-5.84	16.92	-2.86	-70.27	-5.73	-9.12	-42.38
h	.39	-18.24	-.11	-.14	-.15	-2.92	-.03	.04	.01	.01	-.01	-0-	1.96	.17	.26	-7.51
d( )/dt_o																
x	-2.51	17.25	.18	.23	.84	1.85	.05	-.20	-0-	-0-	.02	-.01	-1.24	-.16	-.43	-4.78
k	-.23	25.55	.26	.34	1.25	13.89	.39	-1.51	.16	.17	-.78	.50	-.57	-.07	-.20	-4.78
w	-1.08	-15.98	-.17	-.21	-.78	.72	.02	-.08	.47	.51	-2.38	1.52	-1.27	-.17	-.44	-4.78
t	.50	-17.41	-.18	-.23	-.85	-1.86	-.06	.20	-0-	-0-	-.02	.01	1.25	.16	.43	-4.79
y	-3.01	11.52	.12	.15	.56	1.23	.03	-.14	-0-	-0-	.01	-.01	-.83	-.11	-.29	-4.78
z	-.35	-41.47	-.43	-.55	-2.03	-4.31	-.12	.47	1.36	1.47	-6.81	4.35	-5.34	-.69	-1.84	-4.78
c	-2.28	16.41	.17	.22	.80	.63	.02	-.07	-.02	-.02	.10	-.06	-1.31	-.17	-.45	-4.78
i	-.23	25.55	.26	.34	1.25	13.89	.39	-1.51	.16	.17	-.78	.50	-.57	-.07	-.20	-4.78
r	.000048	-41.47	-.44	-.56	-2.03	-4.30	-.13	.46	1.37	1.47	-6.81	4.34	-5.36	-.70	-1.84	-4.78
r'	.000367	-67.48	-.45	-.65	-3.06	-7.58	.03	1.10	-.72	-.80	5.12	-2.86	-7.68	-.78	-2.48	-4.55
h	-.50	-17.41	-.18	-.23	-.85	-1.86	-.16	.20	-0-	-0-	-.02	.01	1.25	.16	.43	-4.79
d( )/dB																
x	-1.24	11.52	1.26	1.61	3.79	1.85	.37	-.90	-.01	-.01	.12	-.05	-1.24	-.32	-3.02	-4.78
k	-.09	22.06	2.42	3.10	7.26	17.97	3.60	-8.78	.20	.43	-7.10	2.88	-.74	-.18	-1.79	-6.19
w	-1.06	-5.37	-.59	-.75	-1.76	.36	.07	-.18	.24	.51	-8.46	3.43	-.64	-.17	-1.56	-2.40
t	-.25	11.52	1.26	1.61	3.79	1.84	.37	-.91	-.01	-.01	.12	-.05	-1.24	-.32	-3.02	4.74
y	-1.00	11.52	1.26	1.61	3.79	1.85	.37	-.90	-.01	-.01	.12	-.05	-1.24	-.32	-3.02	-7.16
z	-1.22	-3.91	-.43	-.55	-1.29	-.61	-.12	.30	.19	.41	-6.81	2.76	-.76	-.20	-1.84	-.68
c	-1.16	10.71	1.17	1.50	3.52	.61	.12	-.30	-.02	-.04	.68	-.27	-1.28	-.33	-3.11	-4.68
i	-.09	22.06	2.42	3.10	7.26	17.97	3.60	-8.78	.20	.43	-7.10	2.88	-.74	-.18	-1.79	-6.19
r	.000170	-3.92	-.43	-.61	-1.29	-.61	-.12	.29	.19	.38	-6.81	2.75	-.76	-.20	-1.85	-.68
r'	.000817	-9.82	-.85	-1.16	-3.06	-1.49	-.10	1.10	.03	-.21	7.93	-2.86	-1.52	-.21	-4.06	-.82
h	-.75	-3.82	-.42	-.53	1.26	-.61	-.12	.30	-0-	-0-	-.04	.02	.41	.11	1.00	-1.57

1/ ∂H/∂r' and ∂z/∂r are not shown because of errors in the specification of these parts in the computer program.

2/ (∓) indicates the multiplier turned from negative to positive in sign.

Table XI

Critical Parameters of Christ Model

(Per cent change in multiplier associated with

10 per cent change in parameter is in parens)

	1	2	3	4
dx/dg	$\partial c/\partial y$ (11.52)	$\partial t/\partial x$ (-4.78)	$\partial i/\partial x$ (1.85)	$\partial H/\partial x$ (-1.24)
dc/dg	$\partial c/\partial y$ (17.34)	$\partial t/\partial x$ (-7.57)	$\partial H/\partial x$ (-2.07)	$\partial i/\partial x$ (.99)
di/dg	$\partial c/\partial y$ (15.70)	$\partial i/\partial x$ (12.78)	$\partial t/\partial x$ (-4.40)	$\partial H/\partial x$ (-.52)
dH/dg	$\partial c/\partial y$ (-18.24)	$\partial t/\partial x$ (-7.51)	$\partial i/\partial x$ (-2.92)	$\partial H/\partial x$ (1.96)
dx/dt_o	$\partial c/\partial y$ (17.25)	$\partial t/\partial x$ (4.78)	$\partial i/\partial x$ (1.85)	$\partial H/\partial x$ (-1.24)
dc/dt_o	$\partial c/\partial y$ (16.41)	$\partial t/\partial x$ (-4.78)	$\partial H/\partial x$ (-1.31)	$\partial c/\partial r$ (.80)
di/dt_o	$\partial c/\partial y$ (25.55)	$\partial i/\partial x$ (13.89)	$\partial t/\partial x$ (-4.78)	$\partial i/\partial r'$ (-1.51)
dH/dt_o	$\partial c/\partial y$ (-17.41)	$\partial t/\partial x$ (-4.79)	$\partial i/\partial x$ (-1.86)	$\partial H/\partial x$ (1.25)
dx/dB	$\partial c/\partial y$ (11.52)	$\partial t/\partial x$ (-4.78)	$\partial c/\partial r'$ (-3.79)	$\partial H/\partial r$ (-3.02)
dc/dB	$\partial c/\partial y$ (10.71)	$\partial t/\partial x$ (4.68)	$\partial c/\partial r'$ (3.52)	$\partial H/\partial r$ (-3.11)
di/dB	$\partial c/\partial y$ (22.06)	$\partial i/\partial x$ (17.97)	$\partial i/\partial r'$ (-8.78)	$\partial c/\partial r'$ (7.26)
dH/dB	$\partial c/\partial y$ (-3.82)	$\partial t/\partial x$ (-1.57)	$\partial c/\partial r'$ (1.26)	$\partial H/\partial r$ (1.00)

According to this table, $\partial c/\partial y$, $\partial t/\partial x$ and $\partial i/\partial x$ are the most important parameters, having the greatest effect on the critical multipliers. This information is of value in comparing the assumed parameters of the Christ model with estimates presented in this paper, and also becomes relevant when the parameters are viewed in relation to their standard errors. It should be noted, however, that this analysis is of limited value if the parameters show wide variation over the cycle. Thus a given parameter may be critical if it varies greatly over time, and, by large variation, exerts a substantial effect on the multiplier, even if its effect (as shown in Table XI) for a given percentage change is small. On the other hand, a high degree of sensitivity of a multiplier to a change in a given parameter does not necessarily cast doubt upon the reliability of that multiplier if the parameter is stable over time.

F. General Comment on Christ Model

Before proceeding to the parameter estimates made in this study, a few final comments are offered. First, it should be noted that the model includes only policy variables as exogenous. Thus the model is of use only in examining effects of various policy combinations, and is of little or no value for forecasting. A forecasting model requires other exogenous variables in addition to policy variables to explain movements in NNP, interest rates, and other relevant endogenous variables. The policymaker needs information as to the effects of other exogenous variables as well, in order to intelligently design policy. The unexplained discrepancies are sufficiently large to cause a policy to be a failure even with "perfect information" from a model like Christ's (see Table XII).

Table XII

Actual Changes and Predicted Changes from Christ Model: 1965 to 1966

(Assuming Model with ΔB Endogenous)

Variable	Actual Change	Predicted Change
x	37.6	57.2
c	20.8	40.0
i	4.9	5.3
r	0.45	- .0003
B	12.6	-10.1

It is in this spirit that an attempt is made to develop and estimate a model which embodies Christ's considerations with regard to the interdependence of monetary and fiscal actions.

III. Interdependence and Effects of Fiscal and Monetary Actions: Some Empirical Results

In this section, a structural model of a closed economy is presented and the parameters of the model are estimated. Quarterly data for the period 1952-I to 1968-I provide the basic set of observations. These parameter estimates provide a rough check on the assumed parameters used by Christ. Standard errors are calculated to understand better the reliability of the parameters.

Attention is focused on the alternative means of financing government expenditures, and simulations over several quarters yield alternative paths for the endogenous variables, depending on the means of financing. Such experiments provide a basis for comparing the relative effects of taxation, debt expansion and monetary expansion. The impact of a pure monetary action, i.e., swapping high-powered money for government bonds, is also examined, though there is no basis for direct comparison with fiscal actions in the model used here.

A. Structural Model

The model used as a basis for estimation is a modification of that summarized in Section II above. The main difference is that the model in this section is constructed on dynamic principles of behavior rather than within a static framework. This modification is done by introducing Koyck distributed lags, i.e., adjustment mechanisms whereby economic units move toward equilibrium.

The model is not original but is presented as an abbreviated approximation of conventional macroeconometric models. The intent is to examine such models for purposes of determining the effect of alternative means of financing government expenditures. By keeping the model relatively small, the factors at work in producing differential effects can be more readily identified.

Price expectations are considered explicitly in the model and enter through the relationship of nominal and real interest rates. If prices are expected to rise in the future, the relative value of real assets is expected to be unchanged as their prices rise with the general price level. Financial assets, on the other hand, are expected to decline in real value, as their claims on real assets decline.

In this situation, lenders of funds would demand higher interest rates for all loans, sufficient to compensate them for the loss of purchasing power they feel they will suffer due to rising price of real assets over the term of loan. Symmetrically, borrowers would be willing to pay higher rates for all loans, to a degree equal to the increase in real wealth they anticipate will be generated by higher prices of real goods. The nominal rate of interest is defined as

$$r_t = r_t^o + \rho_t,$$

where r_t: nominal rate of interest;

r_t^o: real rate of return;

ρ_t: expected rate of change of prices (considered exogenous).

The determination of actual prices is not considered explicitly in this model. However, due to specification of the interest rate equation, the system will be in equilibrium only when the expected rate of change is equal to the actual rate. Thus for $\rho_t > 0$, at equilibrium, the economy will experience inflation equal to ρ_t. Expectations are determined exogenously and are included only to investigate the possible influence which they might have on the effectiveness of stabilization actions.

<u>Consumption function</u>. Considerable empirical work has been done on the consumption function, and the specification of this model reflects those results.

$$(12) \quad C_t = (1-\beta_1)\,[c_o + c_1 Y_t^d + c_2 r_t^o + c_3 W_{t-1}] + \beta_1 C_{t-1}$$

where C_t: real consumption in period t;

Y_t^d: real disposable income in period t;

W_{t-1}: real wealth in period t-1;

C_{t-1}: real consumption in period t-1.

Expected signs are:

$$c_1 > 0;$$

$$c_2 < 0;$$

$$c_3 > 0;$$

$$1 > \beta_1 > 0.$$

Disposable income is a standard argument in consumption functions. Including the real interest rate is somewhat unique, however, but enables real consumption to reflect the effects of changes in price expectations. The wealth variable is for nonhuman wealth only, implying that permanent income or some measure of human wealth is omitted. However, defining permanent income as Friedman does, this effect is picked up to some extent by the lagged consumption term. 16/

Investment function. Unlike the consumption function, there is no consensus with regard to the arguments in the investment function. The specification used here is analogous to the stock-adjustment model. It is not the same, however, in that a discrepancy between the flow of actual and desired investment is posited. With this specification, a necessary condition for equilibrium in the system is that the flow of net investment be constant, which allows for the stock of real wealth to change continuously.

16/ Milton Friedman, A Theory of the Consumption Function (Princeton, N.J.: Princeton University Press, 1959). See also Marc Nerlove, Distributed Lags and Demand Analysis for Agricultural and Other Commodities (Washington D.C.: U.S. Government Printing Office, June 1958).

(13) $I_t = (1-\beta_2)\ [i_o + i_1X_t + i_2r_t^o] + \beta_2I_{t-1}$

where I_t: real net investment in period t;

X_t: real net national product in period t;

I_{t-1}: real net investment in period t-1.

Expected signs are:

$$i_1 > 0;$$
$$i_2 < 0;$$
$$1 > \beta_2 > 0.$$

The expected signs for X and r^o are familiar, with price expectations again entering the function indirectly via r^o The coefficient on lagged investment is an adjustment coefficient, and therefore expected to be between 0 and 1.

Demand for high-powered money. The demand for high-powered money is specified similar to Christ's, in that this demand function represents the nonbank public's demand for currency and banks' demand for reserves.

(14) $H_t = (1-\beta_3)\ [h_o + h_1X_t + h_2r_t] + \beta_3H_{t-1}$

where H_t: real high-powered money in period t;

H_{t-1}: real high-powered money in period t-1.

Expected signs are:

$$h_1 > 0;$$
$$h_2 < 0;$$
$$1 > \beta_3 > 0.$$

Inclusion of X_t is familiar. Unlike the demand for real goods, the demand for financial assets is posited to be a function of nominal interest rates. An increase in prices will decrease the relative value of cash holdings, and holders of financial assets would thus demand higher interest rates to a degree sufficient to compensate for the expected decline in the value of their cash holdings. The coefficient on H_{t-1} is expected to be between 0 and 1 because of the stock adjustment assumptions underlying the function.

Real NNP definition.

(15) $X_t = C_t + I_t + G_t$

where G_t: real government purchases in period t.

Real disposable income definition.

(16) $Y^d_t = Y_t - T^p_t - T^s_t + R_t$

where T^p_t: real Federal personal taxes;

T^s_t: real state and local taxes;

R_t: real Federal transfer payments.

Real factor income equation.

(17) $Y_t = y_o + y_1 X_t$

where Y_t: real factor income.

Real Federal personal tax function.

(18) $T^p_t = t^p_o + t^p_1 Y_t$

Real state and local tax function.

(19) $T^s_t = t^s_o + t^s_1 X_t$

Real other Federal tax function.

(20) $T^o_t = t^o_o + t^o_1 X_t$

where T^o_t: real other Federal taxes in period t.

Interest rate definition.

(21) $r_t = r^o_t + \rho_t$

where r_t: nominal interest rate in period t;

ρ_t: expected prices in period t.

Real private wealth definition.

(22) $W_t = W_{t-1} + D_t + I_t$

where W_t: real private wealth in period t;

D_t: real Federal deficit in period t.

Real Federal budget identity.

(23) $D_t = G_t + R_t + S_t - T_t^p - T_t^o$

where S_t: real net subsidies plus real net interest in period t.

General comments. The model is similar to that presented by Christ. The budget identity differs from that in Christ's model to the extent that the central bank is not consolidated with the Federal government. Instead the central bank is treated here as a third sector which supplies high-powered money to the system. "Even-keel" operations are approximated by the addition of another equation to the system:

$$D_t = H_t - H_{t-1}$$

where the supply of high-powered money is determined by the size of the Federal deficit (or implicitly, by the amount of Federal spending financed by the issue of debt.)

There are thus eight potential policy variables (G_t, R_t, S_t, t_o^p, t_o^s, t_o^o, D_t, and H_t), any seven of which may be predetermined by the stabilization authorities. Although t_o^s is held to be exogenous, it is further assumed to be beyond the control of both Federal government and the central bank. In addition, t_o^o is included so as to satisfy the NIA budget identity and will not be used as a policy variable. Thus stabilization authorities are here restricted to predetermining any five of the remaining six variables.

The model differs from Christ's in that statistical and institutional factors are added to make the model amenable to estimation. The demand for bonds is not explicit, following the approach of conventional models, though the stock of bonds outstanding is allowed to enter the dynamic solution of the model via the wealth definition.

The variables of the model are classified in Table XIII.

Table XIII

Endogenous Variables	Exogenous Variables
X_t	G_t
C_t	R_t
I_t	S_t
Y_t	t_o^p
Y_t^d	t_o^s
T_t^p	t_o^o
T_t^s	H_t
T_t^o	H_{t-1}
r_t^o	C_{t-1}
r_t	I_{t-1}
W_t	W_{t-1}
D_t	ρ_t

B. Estimated Model: 1952-1968

The model was estimated using two-stage least squares with quarterly data for the period 1952-I to 1968-I. (The data -- observations and definitions -- are given in the Appendix.) The first-stage estimates were run with nine predetermined variables listed above. The intercepts of the tax functions were hypothesized to be equal to zero even though they were estimated originally. The seven behavioral equations were estimated, and the remaining five equations (definitions and identities) were used to derive the reduced form. The estimates are, of course, average relationships for the 1952-1968 period.

Consumption function. The two-stage least squares estimate of the consumption function yielded:

$$(12') \quad C_t = -69.63 + \underset{(1.57)}{.21}\, Y_t^d + \underset{(2.38)}{.10}\, W_{t-1} - \underset{(1.49)}{1.73}\, r_t^o + \underset{(3.68)}{.67}\, C_{t-1}$$

t values are shown in parenthesis below the parameter estimates. All signs are as expected, but the degree of significance of the coefficient of disposable income is quite surprising. The short-run coefficient is quite small relative to that assumed by Christ. In the long run, however, consumption would change by 63 per cent of a short-run change in Y^d, not significantly different from Christ's assumed 70 per cent response.

The coefficient of the interest rate is much smaller than Christ's, although not significant at the 10 per cent level. The coefficient of lagged wealth is of expected sign, and significant. Christ did not include wealth explicitly in his function, however, choosing instead to study the influence of changes in the stock of financial assets through the influence of capital gains on consumption. It is felt that although changes in wealth are generated only by changes in government debt and high-powered money, the wealth variable offered here is superior.

The adjustment coefficient (.67) is satisfying in magnitude, as well as in sign and significance.

Investment function.

$$(13') \quad I_t = -13.51 + \underset{(3.22)}{.10\ X_t} - \underset{(1.55)}{1.87\ r_t^o} + \underset{(6.21)}{.64\ I_{t-1}}$$

All signs are as expected, with the coefficient of r_t^o insignificant at the 10 per cent level. In absolute magnitude (ignoring the t value of r_t^o) the estimated parameters are very close to those assumed by Christ. Again, the adjustment coefficient was consistent with a priori conditions, both in absolute magnitude, and relative to the speed of adjustment of consumption.

Demand for high-powered money.

$$(14') \quad H_t = 2.88 + \underset{(4.52)}{.01\ X_t} - \underset{(3.70)}{.61\ r_t} + \underset{(31.55)}{.91\ H_{t-1}}$$

All variables are significantly different from zero at the 10 per cent confidence level and are of the expected sign. Relative to the Christ model however, the estimated coefficients of X_t and r_t are quite small. The adjustment coefficient suggests that the demand for currency and reserves in the economy adjusts quite rapidly to external shocks.

Factor income definition.

$$(17') \quad Y_t = -22.96 + .86\, X_t$$
$$(114.73)$$

This function is offered, not as representation of the labor market, but instead as a convenient short-cut to determination of disposable income. Its inclusion however is not in contrast to standard procedure employed in other small aggregate demand models.

Tax functions.

$$(18') \quad T^p_t = -.69 + .11\, Y_t$$
$$(31.04)$$

$$(19') \quad T^s_o = -.11 + .04\, X_t$$
$$(67.74)$$

$$(20') \quad T^o_o = -.08 + .21\, X_t$$
$$(39.39)$$

The functions determining state and local taxes and Federal taxes other than personal income taxes were estimated in order to satisfy national income accounting identities and are not to be considered as policy instruments in this paper. The tax functions are rough approximations and represent average relationships for the 1952-1968 period. No attempt is made to allow for changes in marginal tax rates. Generally, however, such changes were infrequent and relatively small during the sample period.

Estimated Elasticities

A direct comparison of parameter estimates from the two studies is difficult due to differences in model specification, data definitions and time period under study. A step toward purification of estimates to enhance comparability is to compute elasticities. The elasticities implied by the results in equations (12') - (14') are shown in Table XIV.

Table XIV

Elasticity	Short-run (One Quarter Impact)	Long-run (Steady State)	Christ
C/Y^d	.23	.69	.78
C/W	.31	.95	---
C/r^o	-.01	-.03	-.05
I/x	.59	1.91	1.20
I/r^o	-.05	-.13	-.10
H/x	.09	.87	1.00
H/r	-.05	-.52	-1.00

Christ's elasticities are apparently for one year, implying that his system, once disturbed, reaches equilibrium within a year. In this sense his assumed elasticities are probably better compared with the long-run or steady-state elasticities generated by the estimated model. Table XIV indicates this is probably true.

The consumption elasticities are smaller for the model of this paper but of the same sign. The elasticity of consumption with respect to current disposable income in the short-run is much lower because the consumption function has lagged consumption as one of its arguments. The long-run elasticity, however, is quite close to Christ's assumed value. The interest rate elasticity is also smaller in both the short and long run; nevertheless, the absolute difference does not appear significant.

Elasticities in the investment function estimated in this paper appear to be consistent with Christ's assumed values. Again, however, there is a tendency for Christ's values to be closer to the long-run values implied by the model estimated here.

The estimated short-run elasticities of high-powered money demand differ quite substantially from Christ's. The income elasticity, in particular, departs most significantly. Although larger, Christ's unitary elasticity assumption is quite close to the long-run elasticity calculated in this model. The interest elasticity also lies below Christ's assumed value in absolute terms. The suggestion that the demand for high-powered money is not equally responsive to changes in income and interest rates has interesting implications in a situation where the monetary authority attempts to use interest rates as a policy goal.

C. Reduced Form Results

The parameter estimates of the seven equations (12') - (14'), (17') - (19') can be combined with the remaining five equations of the system to derive the reduced form of the model. 17/ The estimated reduced form thus reflects all of the differences between the parameters estimated here and those assumed by Christ, as well as differences in model specification. This reduced form provides the matrix of impact multipliers, which can then be used to simulate the effects of various policy actions over time.

Impact Multipliers

The estimated reduced form (government bonds endogenous) implied by the 1952-1967 experience is shown in Table XV.

17/ For purposes of calculating the impact multiplier matrix, wealth was redefined to include only additions of real financial wealth (See Ott and Ott, op. cit.). This change in specification permitted the model to have a steady-state solution without requiring investment to go to zero.

Table XV

Quarterly Impact Multipliers

(Government Bonds Endogenous)

	ΔG	Δt_o	ΔH
ΔX	1.25	-.26	7.39
ΔC	.17	-.24	3.81
ΔI	.08	-.02	3.58
Δr	.02	-.004	-1.52
ΔB	.91	.02	- .70

Multipliers can also be calculated to compare the impact effect of various means of financing government expenditures. [18/] The following results were obtained:

Table XVI

Quarterly Impact Multipliers

	G financed by bond sales and induced taxes	G financed by autonomous taxes and induced taxes	G financed by high-powered money issue and induced taxes
ΔX	1.25	1.06	2.65
Δr	.02	- .01	- .27

These impact multipliers suggest significant influence of changes in high-powered money in the short run, but this observation is contingent on the specification of the model. Additional insight into the quantitative influence of fiscal and monetary actions can be gained by simulating the model over a longer period of time.

Dynamic Simulations

Each of the simulations involved changing a predetermined variable and solving the system successively for several quarters. The

---

18/ The multipliers in Table XVI are linear combinations of those in Table XV, reflecting the government's budget constraint.

experiments that are summarized here are as follows:

(1) $1 billion increase in high-powered money (open market operation);

(2) a $1 billion increase of government spending financed by government bonds and induced taxes;

(3) a $1 billion increase of government spending financed by high-powered money issue and induced taxes;

(4) a $1 billion increase of government spending financed by increases in autonomous taxes and induced taxes (the budget is balanced throughout);

(5) a one per cent increase in expected prices.

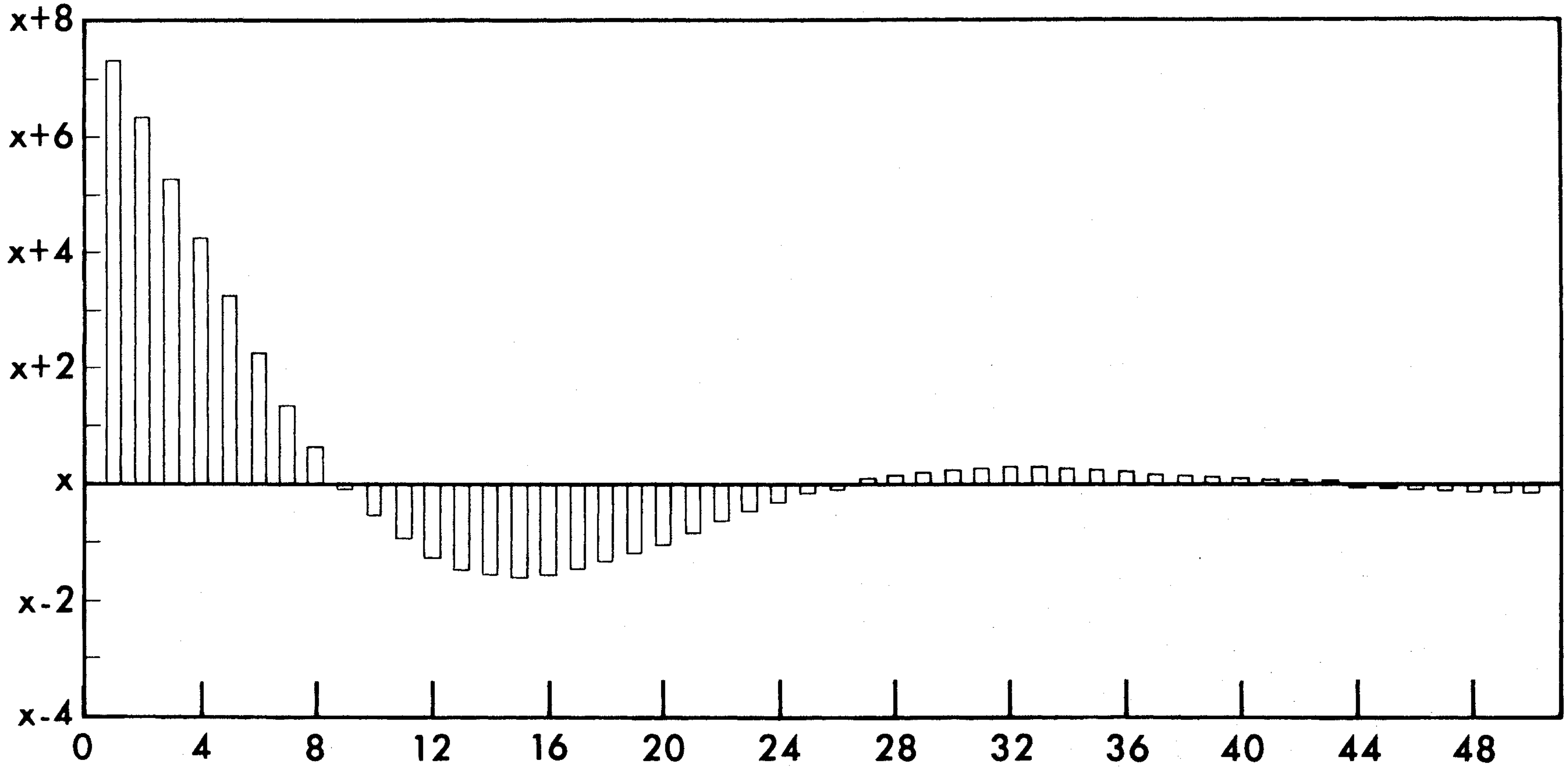
The first case is a once-and-for-all change in high-powered money. Induced taxes are assumed to be used to retire debt, and thus the stock of government bonds outstanding will be reduced, initially, although not by $1 billion. In all following periods, induced taxes are used to retire debt.

Cases (2) - (4) involve the same initial change in government spending followed by a continuous policy of financing that portion of the deficit that is not covered by induced taxes by either issuing government bonds, issuing high-powered money, or increasing autonomous taxes.

<u>Open market purchase</u>. The effect of an open market purchase on X is shown in Chart 1. Initially X increases sharply, but the expansionary effect subsides quickly, reaching zero after about 10 quarters. For a period of about 16 quarters thereafter, X falls

Chart 1

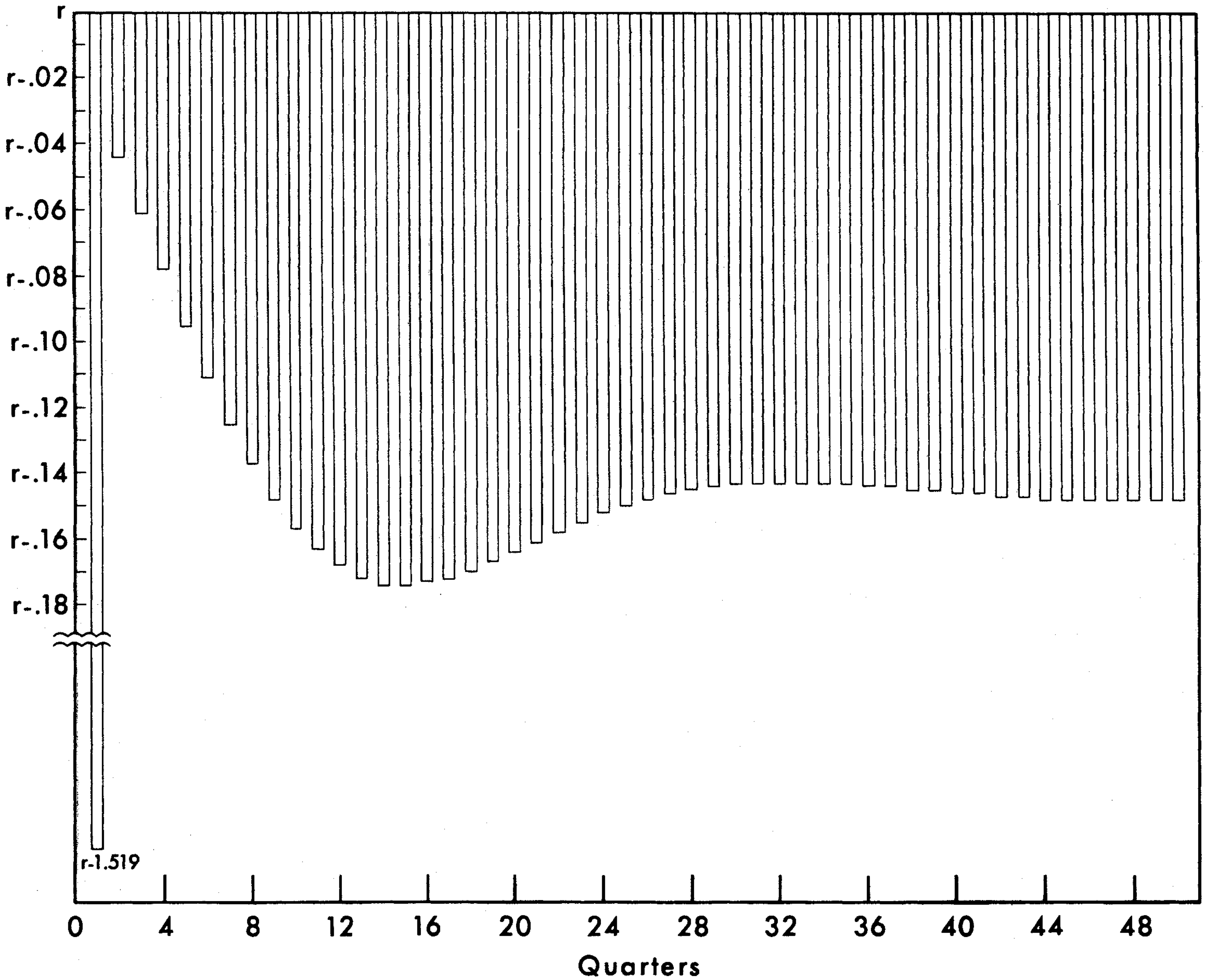
Cumulative Effect on X of an Open Market Operation ($\Delta H_t = 1.0$; $t=1$)



Prepared by Federal Reserve Bank of St. Louis

Chart 2

Cumulative Effect on Interest Rates of an Open Market Operation ($\Delta H_t = 1.0$; t=1)



Prepared by Federal Reserve Bank of St. Louis

below its initial level; then, after having a small positive effect, the effect finally approaches zero after about 50 quarters. In a model of this type, a change in the stock of high-powered money has no permanent effect on income, since at equilibrium the government budget must be in balance and any change in income would result in a budget surplus, due to induced taxes. 19/

There is a permanent effect on the interest rate, however (see Chart 2). Initially, interest rates are pushed down sharply, then rise sharply. After two quarters, the movements are quite small, and, eventually, the interest rate settles to an equilibrium level about 15 basis points below the original level. Since the budget constraint forces income to remain unchanged, interest rates must fall if the private sector is to absorb the increased stock of high-powered money at the same level of income.

Government spending increases. The three cases involving an increase of government spending can be analyzed together, and are summarized in Chart 3.

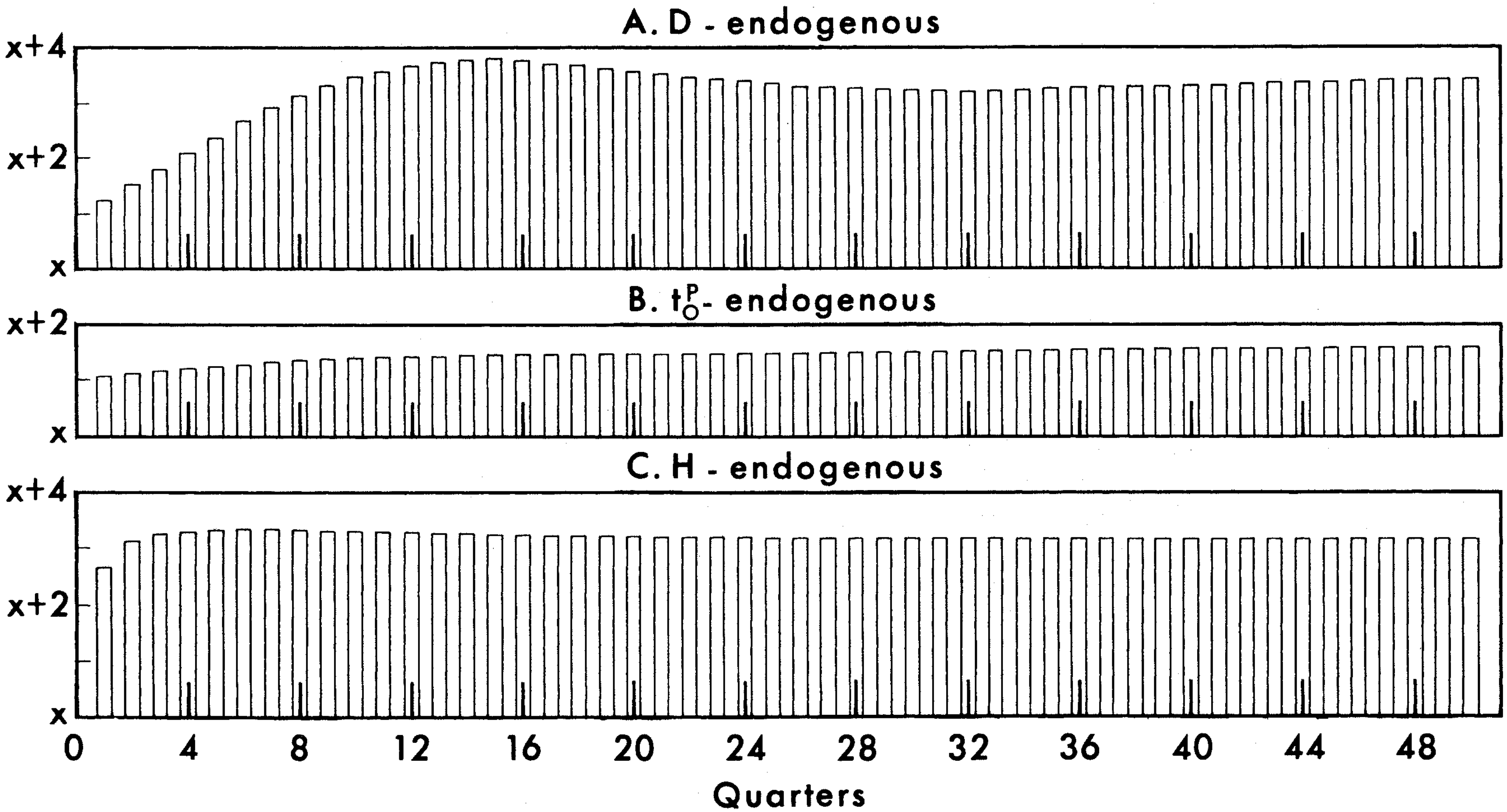
The first case with bond financing shows a steady rise in X for about 14 quarters, followed by a slight decline. X then stabilizes, after about 35 quarters at a level \$3.28 billion above the original. This pattern follows from the interest rate effect of bond-financed government spending (see Chart 4). Interest rates rise with income, but as the multiplier effect gets worked out, the interest

---

19/ Ott and Ott, op. cit.

Chart 3

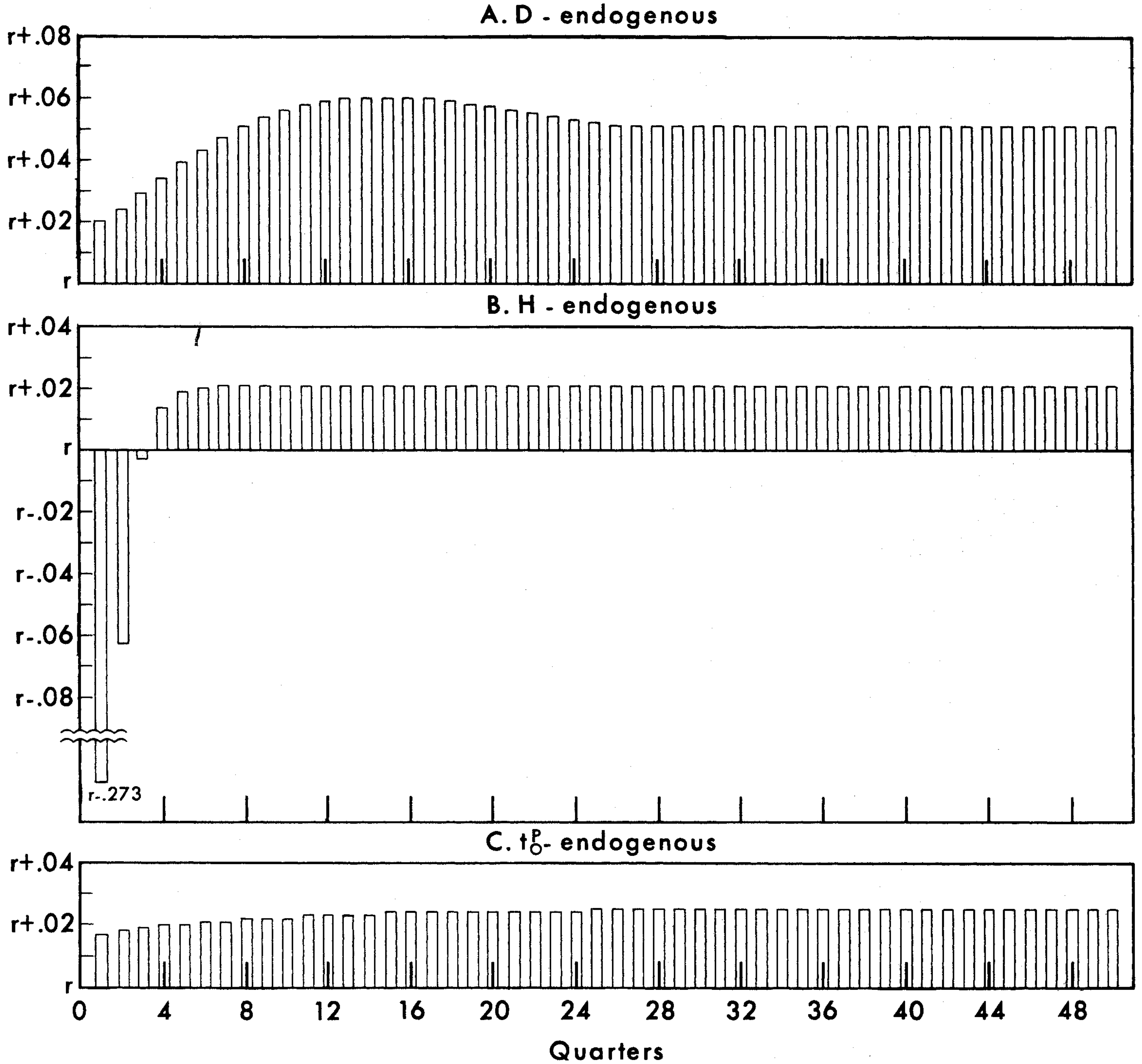
Cumulative Effect on X of a Once and for All Increase in G ($\Delta G_t = 1.0;\ t=1$)



Prepared by Federal Reserve Bank of St. Louis

Chart 4

Cumulative Effect on Interest Rates of a Once and for All Increase in G (ΔG_t=1.0; t=1)



Prepared by Federal Reserve Bank of St. Louis

rate effect eventually overcomes it, leading to declines in GNP from about the 15th to the 32nd quarter. Interest rates end up higher than the original, but only by about 5 basis points.

The second case with high-powered money issue has a much faster impact but eventually stabilizes with X at a level $3.28 billion higher than the original. The main difference between this case and the bond-financing case is that most of the ultimate impact is completed after 3 quarters. Interest rates also stabilize much faster, although not as rapidly as income. Interest rates fall sharply initially, then climb sharply to 2 basis points above the original after about 7 quarters, then stabilize.

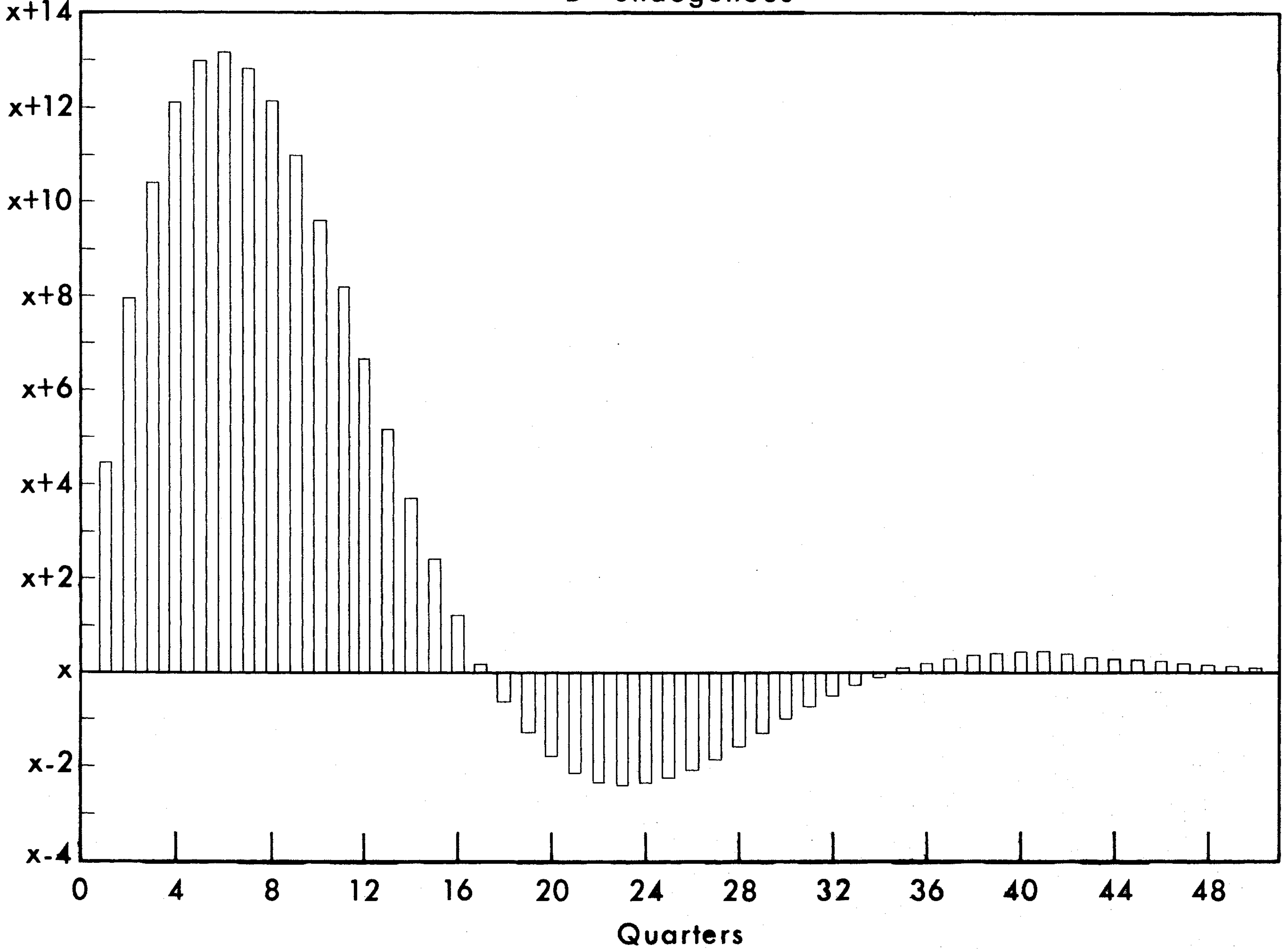
The third case with changes in autonomous taxes tends to behave like the high-powered money case, though with a much less pronounced effect. Income rises sharply in the first quarter, then slowly for about 25 quarters, then stabilizes at a level $1.5 billion above the original. The impact on interest rates is similar, rising sharply, then eventually stabilizing at a level about 2 basis points above the original.

Increase in expected prices. Price expectations play an interesting role in the model in that when the private sector expects prices to rise, private spending can be expected to increase in the short run (Chart 5). This suggests that in addition to considerations of financing influence, stabilization authorities must be aware of other factors which might alter this influence of stabilization actions on the economy. If, for example, rising prices generate expectations of further increases in the future, the affects of anti-

Chart 5

Cumulative Effect on X of a Once and for All Increase in ρ(ρ_t=1.0; t=1)

D - endogenous



Prepared by Federal Reserve Bank of St. Louis

inflationary government actions would be tempered somewhat by the expansionary effect of the increase in price expectations. The influence of expectations suggests that the influence of stabilization actions might vary over the business cycle, and offer problems for future research.

Summary

The dynamic simulations indicate that the time period is of critical importance in evaluating the quantitative influence of monetary and fiscal actions. Different conclusions emerge depending on the length of time period considered in assessing the impact of a policy change. The importance of time is demonstrated more cogently in the following two tables:

Table XVII

Government Spending Multipliers

		1st Qtr.	4th Qtr.	8th Qtr.	Steady State	Christ
ΔB Endogenous	Δx	1.24	2.10	3.12	3.28	2.42
	Δr	.02	.03	.05	.05	.006
ΔH Endogenous	Δx	2.65	3.28	3.29	3.28	3.06
	Δr	- .27	.01	.02	.02	- .002
Δt_o Endogenous	Δx	1.06	1.21	1.33	1.50	1.11
	Δr	.02	.02	.02	.02	.001

Table XVIII

High-Powered Money Multiplier

		1st Qtr.	4th Qtr.	8th Qtr.	Steady State	Christ
ΔB Endogenous	Δx	7.32	4.33	.63	0	1.66
	Δr	-1.52	- .08	- .14	- .15	- .02

These results suggest that any studies that focus on either extreme, one-quarter impact or steady state, have to be interpreted with extreme caution. Short-run stabilization policy is concerned with some interim period, and probably the impact within two years is most important. In the very short run for this model, monetary actions, as reflected in high-powered money, have very powerful effects. Fiscal actions are also important, but the magnitude of the impact is substantially less than that of monetary actions, especially in the first year.

IV. Conclusions

This study has been concerned with the relative influence of monetary and fiscal actions. The literature was reviewed briefly, with primary emphasis placed on Professor Christ's recent work with government budget identities. Christ's work has revived interest in an old issue, the effects of financing of government spending. His model is sufficiently general, however, to encompass some elementary analysis of monetary actions. Christ's work provided the stepping-stone for construction of a simple dynamic model that was explored in a similar manner. The simple dynamic model of 12 equations yielded some preliminary estimates of the time paths of some key economic variables when certain policy actions were taken. The model is only a short step removed from the simplest Keynesian model. Nevertheless, by performing certain experiments, it was found to be rich in implications if not in conclusions.

Despite the fact that time paths were derived, yielding additional information about the impact of monetary and fiscal actions over time, it is still difficult to be specific in recommending stabilization strategy. The model is small by modern-day standards, but still large enough to cause difficulty in understanding. It is the contention of the authors, however, that there is more potential gain from small models that gloss over institutional features than from large models that, almost by definition, defy comprehension. [20] Interestingly, the results are in general consistent with those from larger models, suggesting that perhaps some of the most important relationships have been included. The marginal gain from model expansion may be quite small.

20/ Small models can be subjected to a rigorous examination of identifiability and stability conditions, which are given short shrift in this paper. Consequently, all estimates are subject to question until the model has been tested rigorously. For an example of this type of methodology, see R.L. Basmann, "Remarks Concerning the Application of Exact Finite Sample Distribution Functions of GCL Estimators in Econometric Statistical Inference," *Journal of the American Statistical Association* (December, 1963), pp. 943-76.

APPENDIX

Data and Sources

Yields are in percentages, i.e., 3.5 per cent. All adjustment from nominal to real terms was accomplished by use of the GNP deflator (1958 = 100).

X = real net national product (national income accounts)

C = real consumption expenditures (national income accounts)

I = real net investment, includes state and local government purchases of goods and services (national income accounts)

G = real Federal government purchases of goods and services (national income accounts)

R = real Federal transfer payments (national income accounts)

S = real net Federal subsidies plus real interest payments (national income accounts)

T^p = real personal tax and non-tax payments to the Federal government (national income accounts)

T^s = real state and local taxes (total real personal tax and non-tax payments -- minus those made to the Federal government -- national income accounts)

T^o = other real Federal taxes (total real Federal receipts minus real personal tax and nontax receipts -- national income accounts)

H = real stock of high-powered money (real total member bank reserves plus currency outstanding -- Federal Reserve Bank of St. Louis)

Y^d = real disposable personal income (national income accounts)

Y = real factor income (real personal income minus real transfer payments to persons -- national income accounts)

r = nominal rate of interest (Moody's yield on corporate Aaa bonds)

ρ = expected rate of change of prices (average rate of change of the implicit GNP deflator over the previous twenty quarters -- national income accounts)

r^o = real rate of interest $(r-\rho)$

W = real private wealth. Series constructed by totaling real high-powered money stock, real private holdings of U.S. Government debt, and real physical capital. Series of real physical capital constructed by accumulating net private investment plus the stock of capital in the fourth quarter of 1951 (Goldsmith series adjusted by fixed investment deflator.

DATA

Quarter	X	C	I	Y	Y^d	r^o	r
1952 1							
2	362.9	238.1	57.4	284.9	260.2	- .67	2.93
3	367.5	239.1	60.3	290.2	265.9	- .56	2.95
4	378.3	246.8	65.5	294.4	269.5	- .32	2.99
1953 1	384.5	250.1	64.1	298.5	272.9	.20	3.07
2	387.8	251.5	64.0	302.3	276.9	.75	3.32
3	384.3	251.1	61.1	300.6	275.9	.98	3.27
4	377.9	250.4	54.1	300.1	276.1	1.18	3.13
1954 1	372.4	250.8	56.0	295.8	275.7	.93	2.96
2	371.1	253.3	56.9	294.2	275.1	.45	2.88
3	375.3	256.9	59.7	297.3	278.5	.28	2.88
4	383.4	261.9	64.4	302.9	284.3	.23	2.89
1955 1	394.8	267.6	70.7	307.1	287.5	.25	2.96
2	401.1	273.0	75.0	314.9	294.4	.09	3.03
3	407.1	276.3	75.9	321.5	300.2	.24	3.10
4	410.5	279.9	77.3	327.0	304.5	.64	3.12
1956 1	407.1	279.9	74.7	329.1	306.0	.87	3.10
2	409.3	280.3	73.7	332.4	308.3	1.45	3.26
3	408.5	280.8	73.8	333.1	309.1	1.58	3.42
4	413.9	284.7	73.1	338.7	314.0	1.62	3.68
1957 1	416.3	286.6	70.7	338.5	314.2	1.73	3.70
2	415.3	287.0	69.9	339.5	315.9	1.59	3.77
3	417.0	289.3	71.0	341.1	317.6	1.84	4.07
4	409.1	289.7	64.2	338.1	316.4	1.72	4.00
1958 1	398.8	285.6	58.4	332.8	313.5	1.36	3.61
2	400.9	287.5	57.4	331.5	314.5	1.19	3.58
3	412.0	291.9	63.3	339.3	321.5	1.38	3.87
4	422.5	295.2	70.7	344.4	325.7	1.55	4.09
1959 1	429.4	302.3	73.7	349.2	329.2	1.49	4.13
2	439.2	307.0	80.8	356.1	334.9	1.87	4.35
3	434.1	309.9	71.9	354.1	332.6	1.93	4.47
4	438.5	310.0	75.7	356.9	335.4	1.93	4.57
1960 1	448.0	313.8	80.4	362.4	338.8	1.94	4.55
2	447.4	317.7	75.2	365.3	341.2	1.83	4.45
3	445.2	316.4	72.7	365.4	341.8	1.68	4.31
4	442.2	316.4	67.5	362.4	339.5	1.70	4.32
1961 1	440.6	316.2	65.1	363.0	341.8	1.69	4.27
2	449.7	320.4	69.6	368.6	347.7	1.86	4.28
3	458.0	323.9	73.5	373.6	352.8	2.15	4.44
4	467.6	329.5	77.1	381.5	359.6	2.38	4.41
1962 1	472.6	333.3	77.0	385.4	362.6	2.44	4.41
2	480.6	335.7	78.7	391.6	366.8	2.47	4.30
3	486.0	340.1	80.1	394.2	368.5	2.63	4.34
4	490.3	344.6	80.2	397.2	371.1	2.70	4.26
1963 1	493.0	348.5	79.2	401.0	375.7	2.65	4.20
2	497.0	350.9	80.6	404.5	378.0	2.76	4.22
3	505.4	356.1	83.4	409.7	383.1	2.86	4.29
4	512.1	357.7	87.6	414.8	388.1	2.94	4.33
1964 1	520.2	366.3	85.6	421.0	396.5	2.96	4.37
2	527.5	370.7	88.4	427.6	406.2	3.03	4.41
3	533.8	378.6	88.4	435.8	412.6	3.05	4.41
4	536.3	379.3	91.3	441.6	417.1	3.04	4.43

DATA

Quarter	X	C	I	Y	Y^d	r^o	r
1965 1	548.6	387.9	98.2	447.7	421.3	2.98	4.42
2	556.9	393.4	98.3	455.5	427.1	2.99	4.44
3	568.2	400.3	101.6	464.8	441.1	3.06	4.50
4	581.8	409.2	105.8	476.1	449.1	3.20	4.61
1966 1	593.1	415.7	108.9	482.2	455.1	3.40	4.81
2	597.4	414.8	112.3	486.8	454.6	3.51	5.00
3	603.0	420.0	110.9	493.4	461.4	3.68	5.32
4	609.6	420.6	116.3	497.5	466.6	3.59	5.38
1967 1	607.8	424.8	105.8	501.1	471.9	3.26	5.12
2	610.5	431.2	100.2	504.3	476.3	3.34	5.26
3	616.2	431.8	104.3	509.8	479.5	3.61	5.62
4	622.3	434.1	109.6	514.7	483.7	3.87	6.03
1968 1	632.4	444.9	108.4	522.4	491.8	3.86	6.13
2							
3							
4							

DATA

Quarter	G	H	R	ρ	T^p	T^s	T^o	W
1952 1								903.3
2	63.1	52.9	12.9	3.61	34.3	3.3	70.3	907.1
3	66.6	53.6	13.7	3.51	34.5	3.5	70.2	908.0
4	65.6	53.9	13.7	3.31	35.1	3.5	73.0	912.2
1953 1	68.4	53.9	13.7	2.87	35.6	3.6	74.9	920.7
2	70.7	53.9	13.7	2.57	35.4	3.6	75.0	931.7
3	70.0	54.7	13.9	2.29	34.9	3.7	73.2	941.2
4	70.8	54.7	14.5	1.95	34.8	3.7	67.6	946.6
1954 1	62.6	53.9	15.2	2.02	31.4	3.9	64.0	939.5
2	57.1	53.8	16.0	2.43	31.1	4.0	63.9	945.5
3	54.6	54.2	16.5	2.60	31.2	4.0	64.8	952.9
4	52.7	54.5	17.2	2.66	31.7	4.0	67.2	958.0
1955 1	51.5	54.2	17.1	2.71	32.4	4.4	70.3	961.8
2	49.9	54.2	17.3	2.94	33.4	4.4	72.4	970.0
3	51.3	54.2	17.4	2.86	34.3	4.4	74.5	976.0
4	50.3	54.0	17.4	2.48	35.4	4.5	76.1	980.7
1956 1	50.0	53.5	17.8	2.23	36.1	4.9	75.9	979.6
2	50.3	53.2	17.9	1.82	37.1	4.9	77.0	977.6
3	48.7	52.7	18.3	1.84	37.3	4.9	76.1	973.4
4	49.8	52.5	18.4	2.06	38.0	5.0	78.6	974.1
1957 1	52.1	52.0	19.1	1.96	38.3	5.2	80.0	975.0
2	52.2	51.8	20.3	2.18	38.6	5.2	79.2	973.3
3	51.3	51.3	20.4	2.23	38.5	5.3	78.6	974.1
4	51.3	51.0	21.6	2.28	37.8	5.4	75.2	974.8
1958 1	52.2	50.8	22.8	2.24	36.6	5.5	70.8	972.4
2	53.4	51.2	24.5	2.38	36.0	5.5	70.4	975.7
3	53.9	51.3	24.9	2.48	37.1	5.6	73.8	977.5
4	53.9	51.2	24.4	2.54	37.4	5.7	77.2	982.2
1959 1	53.5	51.3	24.6	2.64	38.4	6.1	80.9	988.5
2	52.6	51.4	24.3	2.48	39.3	6.1	84.2	966.8
3	51.9	51.2	24.3	2.54	39.6	6.2	82.3	1002.5
4	51.9	51.1	25.2	2.64	40.4	6.4	82.2	1010.3
1960 1	51.2	50.9	25.3	2.61	41.9	6.9	88.4	1014.6
2	51.0	50.6	25.4	2.62	42.5	7.0	88.0	1016.7
3	51.8	50.5	26.1	2.63	42.6	7.1	85.8	1018.2
4	51.8	50.7	26.9	2.62	42.6	7.2	84.6	1018.2
1961 1	52.2	50.9	28.7	2.58	42.6	7.3	83.6	1019.8
2	54.2	50.8	29.2	2.42	42.7	7.4	86.2	1026.3
3	55.9	51.1	29.7	2.28	43.1	7.4	87.9	1037.0
4	55.9	51.6	29.3	2.02	43.7	7.5	90.8	1039.6
1962 1	58.6	51.8	29.8	1.97	44.5	8.1	90.8	1045.8
2	60.7	52.2	29.4	1.83	45.9	8.3	92.6	1055.8
3	60.2	52.3	29.6	1.70	47.0	8.4	94.1	1061.4
4	60.6	52.6	30.4	1.56	48.0	8.5	95.2	1065.6
1963 1	60.8	53.0	31.7	1.54	48.4	8.7	97.3	1070.5
2	59.0	53.4	30.7	1.46	48.4	8.8	98.7	1077.4
3	59.6	53.8	30.8	1.43	48.4	8.9	99.3	1082.8
4	58.7	54.3	31.1	1.39	48.7	9.1	100.7	1087.5
1964 1	58.5	54.7	32.4	1.40	47.1	9.6	98.2	1094.2
2	59.3	55.1	31.6	1.38	43.1	10.0	94.6	1100.5
3	57.8	55.5	31.8	1.36	44.7	10.2	97.2	1105.0
4	56.7	56.1	31.9	1.38	46.1	10.4	98.3	1110.3

DATA

Quarter	G	H	R	ρ	T^p	T^s	T^o	W
1965 1	56.4	56.3	33.3	1.44	49.0	10.6	103.2	1116.3
2	57.2	56.7	32.4	1.45	50.0	10.8	104.0	1120.6
3	58.1	57.2	36.1	1.44	48.9	11.0	102.2	1126.1
4	59.6	57.9	34.8	1.42	49.9	11.2	105.4	1134.6
1966 1	61.8	58.2	35.9	1.41	52.3	11.6	112.6	1141.0
2	64.0	58.5	35.1	1.50	55.1	12.1	115.7	1144.4
3	66.9	58.5	36.7	1.64	56.3	12.4	117.8	1148.4
4	67.9	58.2	39.3	1.80	57.6	12.7	118.5	1152.4
1967 1	72.7	58.7	42.0	1.86	58.3	12.8	118.0	1160.6
2	75.1	59.2	42.5	1.92	57.3	13.2	117.2	1158.9
3	75.6	59.4	42.6	2.00	59.5	13.4	119.3	1166.2
4	75.6	59.8	43.0	2.16	60.2	13.7	121.4	1170.9
1968 1	78.1	60.1	44.9	2.26	61.6	14.0	128.7	