Tobias F. Rötheli

Tobias F. Rötheli, an economist at the Swiss National Bank in Zürich, was a visiting scholar at the Federal Reserve Bank of St. Louis. Laura A. Prives provided research assistance.

Money Demand and Inflation in Switzerland: An Application of the Pascal Lag Technique

IN 1973, the Swiss National Bank ceased pegging the Swiss franc to the U.S. dollar. In so doing, the Swiss monetary authorities gained control over the domestic money stock. This article describes the role of money demand estimates in the new monetary policy. It then assesses this foundation of policy by developing a statistical model for money demand tailored to the current exchange rate and monetary control regime.

SWISS MONETARY POLICY

Under the Bretton Woods system, Switzerland was one of many countries to experience the transmission of U.S. inflation to its economy. Because the Swiss National Bank pegged the exchange rate of the Swiss franc against the U.S. dollar, there was a close connection between U.S. and Swiss inflation. Arbitrage saw to it that changes in the dollar prices of internationally traded goods were matched by proportional changes in corresponding Swiss franc prices. Competition caused the prices of Swiss domestic goods to keep pace with the prices of internationally traded goods. Meanwhile, the public adjusted the Swiss money stock to the rising price level, in order to hold real money balances at the desired level.[1]

The Determination of Monetary Targets

The beginning of 1973 marked a change in the monetary regime. With the transition to flexible exchange rates, the Swiss monetary authorities

[1]This does not mean that inflation is not a monetary phenomenon. Under fixed exchange rates, inflation in a particular country is not caused by the money growth of that country, but by the combined money growth of all countries participating in this monetary arrangement. In such an environment, a small country is a price taker, with virtually no influence on the world price level. When the exogeneous price level rises, domestic residents restore their real balances by accumulating foreign exchange (dollars) through current account surpluses. These earnings are then converted to domestic currency (Swiss francs) by the central bank (the Swiss National Bank) which is ready to make any transaction at the given exchange rate.

Monetary Targets and Monetary Growth

At the end of 1974, the Swiss National Bank (SNB) announced the first monetary target.[1] Until 1978 M1 targets were used. These targets were translated into operational targets for the monetary base. To accomplish this, a dynamic model forecasting the multiplier (the ratio between M1 and the base) was developed.[2] The policy was implemented mainly through foreign exchange purchases and sales.

The actual course of the money stock did not always follow its announced path. The table at right shows the targeted and effective money growth rates up to 1986. The Swiss National Bank tried, mainly in the seventies, to dampen erratic movements of the exchange rate. In 1978, for example, the Swiss franc appreciated strongly against the dollar; as a result, Swiss exports of goods and services, approximately 40 percent of gross national income, declined. To prevent further appreciation of the Swiss franc, the monetary authority expanded the money supply far beyond the target.[3] The monetary target for 1978 was abandoned in September. There was no target for 1979, and since 1980 targets for the adjusted monetary base have been used.[4]

While exchange rate considerations led to marked deviations from the projected money path, these deviations were neutralized in the long run. On average, over the 11 years for which targets were announced, the annual money growth was only 0.14 percent higher than the targeted value.

Monetary Growth: Targeted and Effective

	Target Variable[1]	Target[2]	Effective[2]
1975	M_1	6	4.4
1976	M_1	6	7.7
1977	M_1	5	5.5
1978	M_1	5	16.2
1979	—	—	—
1980	M_0	4[3]	−0.6[3]
1981	M_0	4	−0.5
1982	M_0	3	2.6
1983	M_0	3	3.6
1984	M_0	3	2.5
1985	M_0	3	2.2
1986	M_0	2	2.0

This table is updated from Kohli and Rich (1986).

[1] M_1 covers currency outside the federal government and the commercial banks, as well as demand deposits of Swiss nonbanks with the postal giro system and the commercial banks. M_0 stands for the adjusted monetary base (deposits of private sector with the SNB and outstanding bank notes less the month-end bulge in SNB credit to the commercial banks).

[2] Average of monthly year-on-year rates of change.

[3] The target for 1980 was defined as the average percentage increase in M_0 over the level of November 1979. For each month of 1980, the percentage increase over the level of November 1979 was calculated. The monthly growth rates were in turn compounded in order to obtain annualized rates. The effective rate of −0.6 percent represents the average of the annualized growth rates.

[1] Kohli and Rich (1986) provide a survey of the Swiss experiment of monetary control. For a comparison of U.S. and Swiss experience with monetary targeting, see Rich (1987).

[2] See Büttler et al (1979) on the multiplier model.

[3] See Niehans (1984), chapters 11 and 13, for an in-depth treatment of the relation between the money supply and real exchange rate fluctuations.

[4] Rich and Béguelin (1985) give detailed information on both M1 and base targeting and reasons for the transition to the latter.

could determine domestic money growth and inflation independently.

Monetary targets played a central role in the implementation of the new policy (see shaded insert above). The Swiss National Bank relied strongly on money demand estimates in establishing those targets. An early econometric study of Swiss money demand was published by Schelbert in 1967. In later research, Vital (1978), Kohli (1985), and Kohli and Rich (1986) pooled data from the fixed and the flexible exchange rate period in their samples. According to these studies the deflated monetary aggregates (from the monetary base to M3) can be well explained by two variables: the interest rate and national income.

The long-run goal of Swiss monetary policy is price level stability. In order to achieve this goal, the nominal money stock has to increase by as much as the growth in the demand for real money balances. This is where the money demand estimates enter the policy-making process. Since interest rate changes are hardly predictable, they are

not taken into consideration when formulating the monetary target. This leaves the income elasticity of money demand as the decisive coefficient. Multiplying the income elasticity by the expected growth of real income provides an estimate of the growth of the money supply consistent with a stable price level.

The statistical findings indicate that the income elasticities of the demand for base money and M1 are close to unity. This leads to the rule of thumb that price level stability can be achieved if money growth is equal to the growth of real income. The growth potential of the Swiss real income is estimated to be around 2 percent per year.[2] Accordingly, the Swiss monetary targets have been gradually lowered over time to 2 percent in 1986.

Monetary Regime and Money Demand Estimation

With the change to flexible exchange rates, the estimation of the money demand function must be reconsidered. Under the current regime of monetary control, the nominal money supply is exogenous. Consequently, price level movements must bring aggregate real money balances in line with the desired level. The Chow money demand specification, the one most widely used in Swiss money demand estimates, does not adequately capture this adjustment process.[3] This problem appears in empirical findings: Heri (1986) reports that the explanatory power of a Chow specification decreases substantially when the sample contains data only from the flexible-exchange-rate period.

The present article develops a model of price level adjustment to estimate Swiss money demand for the period from I/1973 to IV/1986. Since we are interested in accurate measurements of the income and interest rate elasticities, an estimation procedure is chosen that considers a broad range of dynamic adjustments of the price level.

ESTIMATING THE MONEY DEMAND FUNCTION FOR THE FLEXIBLE-EXCHANGE-RATE PERIOD

We start with a long-run demand for money function:

(1) $M_r^d = f(Z)$,

where M_r^d denotes real money balances demanded, and Z is a set of variables, usually including a measure of real income and one or more interest rates. Equilibrium requires that equation 2 holds:

(2) $M/P^* = M_r^d$.

The nominal money stock, M, is exogenous. Therefore, this equation determines the equilibrium price level P^*. The actual price level, P, does not always equal P^*. Laidler (1985) elaborates:

> When a flexible price economy is pushed off its long-run demand-for-money function, it moves back by way of the influence of price level changes on the stock of real balances. If the price level is perfectly flexible, such adjustment is instantaneous, and only a long-run aggregate demand-for-money function is observable. However, if prices move less than instantaneously, we would observe the economy moving slowly to equilibrium over time by way of price level changes influencing the quantity of real balances.[4]

To illustrate this point, consider the following experiment: We start with the price level in equilibrium (at P_0^*). Now, the quantity of money is increased. At the prevailing price level, real balances are above their equilibrium value, which induces people to increase their spending and investment. In an economy close to full employment, the increased demand will drive up prices.

In this process, existing contracts are renegotiated over time. Hence, the price level does not jump to the new equilibrium (P_1^*) immediately; instead, it adjusts gradually. Figure 1a shows two possible shapes of this adjustment process.[5] In its

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[2]For an up-to-date study of the Swiss potential real income growth, see Büttler, Ettlin and Ruoss (1987).

[3]Chow (1966) introduced the following adjustment specification for money demand estimates:

$M_{r,t} - M_{r,t-1} = \beta(M_{r,t}^d - M_{r,t-1}) \quad 0<\beta\leq 1,$

where M_r denotes real money balances, and M_r^d is the long-run money demand. Laidler (1985), pp. 111–12, points out that this "real adjustment" version coincides with a price adjustment version based on the partial-adjustment hypothesis only if the money supply is invariant over time.

[4]Laidler (1985), p. 111.

[5]The notion that the real money stock can deviate from the desired money stock and that this discrepancy is only slowly diminished through price level changes has recently been called the "buffer stock concept" of money. Laidler (1987) gives a description of buffer stock money and the transmission mechanism.

Figure 1a
Adjustment Paths of the Price Level

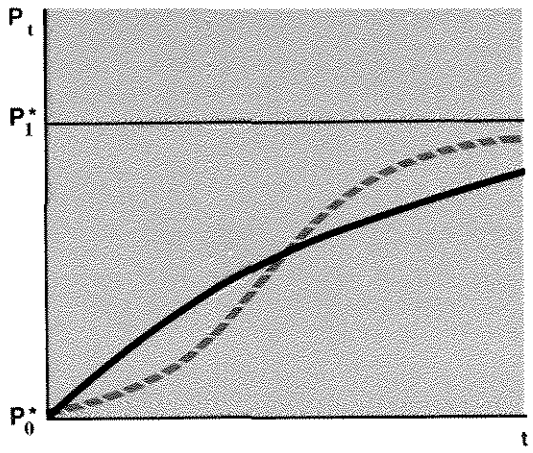
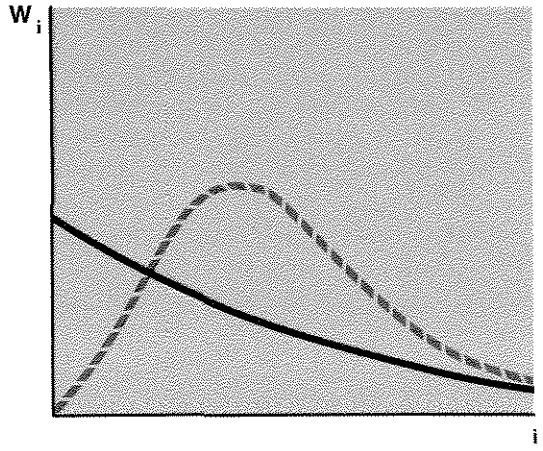


Figure 1b
Lag Weights for the Price Level Adjustment



general form, the adjustment process can be written as

$$(3)\quad P_t = \sum_{i=0}^{\infty} w_i P^*_{t-i},$$

$$\text{with} \quad \sum_{i=0}^{\infty} w_i = 1.$$

Figure 1b shows the pattern of lag weights (w_i) that correspond to the two adjustment paths in figure 1a.[6] The lag weights are also called the speed of adjustment since they measure the adjustment of the price level per unit of time. The Koyck lag, the simplest case, is denoted by the solid line. It can be derived from a partial-adjustment hypothesis which states that the gap between P_t and P_t^* is closed by a constant fraction (β) per unit of time. This implies that the adjustment speed of the price level is highest at the beginning and diminishes steadily thereafter. Because of its simplicity, this adjustment specification has been the most popular form of capturing the dynamic aspects of money demand.[7]

Laidler questions the application of the partial adjustment hypothesis to the behavior of the price level. He writes:

> [W]e would have to argue that it is possible to capture in one simple parameter β the entire transmission mechanism whereby the price level responds to discrepancies between the supply and demand for nominal money. This might be possible, though it seems implausible to say the least.[8]

Laidler concludes that if we are suspicious of the validity of the partial-adjustment hypothesis, then we must also be suspicious of all other parameters (for example, income and interest rate elasticities) estimated with a partial-adjustment specification.

Estimates for Switzerland suggest that the response of the price level to changes in the money stock resembles the pattern shown by the dashed line in figures 1a and 1b.[9] This adjustment is characterized by a slowly increasing adjustment speed in the initial stage of the process; it takes time for the price level movement to build momentum. To get accurate estimates of income and interest rate elasticities, it is therefore worthwhile to consider a

[6]Equations 2 and 3 ensure that a constant growth rate of money eventually leads to a rate of inflation equal to the rate of money growth.

[7]See Thornton (1985) for a concise overview of applications of the partial-adjustment hypothesis in money demand estimates.

[8]Laidler (1985), p. 111.

[9]See Wasserfallen (1985) and Zenger (1985).

wider range of possible lag patterns than just the Koyck specification.

The Pascal Lag Technique

The Pascal lag distribution is a flexible instrument for capturing the dynamic adjustment process discussed above. Solow (1960) suggested that the w_i in equation 3 can be represented by the Pascal distribution. Applied to the case at hand, this specification takes the form:

$$(4)\ P_t = (1-\lambda)^r \sum_{i=0}^{\infty} \binom{r+i-1}{i} \lambda^i P^*_{t-i} + \varepsilon_t,$$

where r is a positive integer, λ is a parameter to be estimated, and ε_t denotes an error term. The combination term in parentheses after the summation sign is a scalar value that depends on r and i. Kmenta (1986) presents an instructive graphical example of how the shape of the lag changes with different values of r.[10] In the simplest case ($r=1$), the Pascal lag reduces to the Koyck lag. Thus, this technique captures a Koyck-type adjustment while opening the possibility of tracing a lag pattern similar to the dashed line in figure 1b.

The Pascal lag is estimated with a maximum likelihood procedure, that searches for the parameter values that minimize the residual sum of squares. It can be estimated in either its autoregressive form or in its distributed lag form.[11] In this study, the distributed lag form is chosen because a possible misspecification of the serial correlation properties of the residual process can lead to flawed parameter estimates in the autoregressive specification.[12] As Maddala and Rao suggest, the order of the Pascal Lag (r) can be chosen by selecting the specification that maximizes the adjusted R^2.

The lag technique used here implies an infinite adjustment process. Because of the finite sample size, this poses a problem. Two ways to deal with this issue shall be briefly described for the simplest case, the geometric lag.[13] Equation 4 can be written as

$$(4')\ P_t = (1-\lambda) \sum_{i=0}^{t-1} \lambda^i P^*_{t-i} + (1-\lambda) \sum_{i=t}^{\infty} \lambda^i P^*_{t-i} + \varepsilon_t.$$

The first part on the right-hand side contains exogenous variables as far back as the sample period runs. The second term contains values that go back to infinity. This second term, however, can be written as

$$(1-\lambda) \sum_{i=t}^{\infty} \lambda^i P^*_{t-i} = \lambda^t E(P_0).$$

Thus, the term $\lambda^t E(P_0)$ can be substituted for the infinite part of the equation. In exchange for avoiding the problem of dealing with an infinite series, however, we face a new problem: the expected value of P_t [$E(P_0)$ for $t=0$] is not observable. This problem is approached in two ways. First, $E(P_0)$ is estimated by including an additional parameter.[14] Second, the actual value of P_0 is used in place of $E(P_0)$. The first procedure shall be called "the method of free parameters;" the latter shall be called "the method of determined parameters."

The Specification of the Model

The specification of the long-run money demand function is

$$(5)\ m_t - p^*_t = \alpha_0 + \alpha_1 y_t + \alpha_2 R_t \quad \alpha_1 > 0,\ \alpha_2 < 0.$$

The estimates are conducted with differenced data. Therefore, no estimate of the constant (α_0) is provided. Lower case letters denote logarithmic variables. The semi-logarithmic specification of money demand is the most widely used in Switzerland. All variables (except the income, y_t) are quarterly averages. The price level is represented by the consumer price index. The money stock variable is M1, and the income variable is the real gross domestic product. R_t is the return (in per-

[10] Kmenta (1986), p. 537.

[11] See Maddala (1977) for a comprehensive treatment of the Pascal lag. Maddala and Rao (1971) give a thorough description of the estimation methods for the Pascal lag model.

[12] See Maddala and Rao p. 84, and Harvey (1985), chapter 7.

[13] Following Maddala, and Maddala and Rao.

[14] With the increasing order of the Pascal lag an increasing number of expected initial values must be replaced by new parameters (see Maddala and Rao, p. 80). That is why a selection criterion ($\bar{R}^2$) is used that is adjusted for the degrees of freedom.

centage points) on three-month Euro-deposits in Swiss francs.[15]

Estimation Results

The first round of estimates is conducted by using the method of free parameters. The equilibrium price level in equation 4 is replaced by its determinants according to equation 5. The following equation is then estimated:

$$(6)\ \Delta p_t = (1-\lambda)^r \sum_{i=0}^{\infty} \binom{r+i-1}{i} \lambda^i (\Delta m_{t-i} - \alpha_1 \Delta y_{t-i} - \alpha_2 \Delta R_{t-i}) + \varepsilon_t.$$

Table 1 contains the $\bar{R}^2$s and the estimates of λ for orders of the Pascal lag ranging from one to four. The λ estimates, which range from 0.77 to 0.97, indicate that the empirical adjustment is rather slow; fast adjustment would imply a λ close to zero. The lag distribution implied by the estimates of λ are shown in chart 1. Since the maximum $\bar{R}^2$ is achieved with the $r=2$ specification, the estimates reject the Koyck lag ($r=1$) as the best specification of the adjustment process. Visible for the optimal case ($r=2$) is a small adjustment of the price level within the quarter of the disturbance. In contrast to the Koyck lag, w_i reaches its maximum value only six quarters after the disturbance and then slowly decreases. It takes approximately 10 quarters for the price level to adjust by 50 percent toward a new equilibrium value. This is much closer to the 12 quarters that both Wasserfallen and Zenger report than the 19 quarters implied by the Koyck estimate. Chart 2, the empirical counterpart to figure 1a, shows the adjustment paths of the price level implied by the Pascal lag estimates for $r=1$ and $r=2$. The chart assumes an increase in the equilibrium price level from one to two in period one. The two adjustment paths show a similar response of the price level only over the first year.

Table 1

$\bar{R}^2$ Statistics and λ Estimates for Pascal Lag Estimates of Different Order (r): I/1973–IV/1986

r	$\bar{R}^2$	λ
1	0.3834	0.9663
2	0.4469	0.8677
3	0.4443	0.8025
4	0.4433	0.7741

Table 2 contains the parameter estimates and various statistics for the $r=2$ case. The two α coefficients have the expected signs. The interest semi-elasticity is very close to the corresponding estimate reported in Kohli and Rich (1986). The income elasticity, however, is substantially smaller than in previous estimates and not significantly different from zero. As the appendix points out, however, using the method of free parameters implicitly involves the estimation of time trends, which can affect the reported results.

The method of determined parameters is used to generate an alternative set of estimates. The $\bar{R}^2$ criterion again leads to the choice of the $r=2$ case as the best dynamic specification; the results are presented in table 3. The income coefficient is 0.83; this time, it is significantly different from zero. The other estimated coefficients are very close to the estimates obtained from the first method.

The method of determined parameters depends heavily on the starting values of the sample period. The model is correctly specified only when the initial values of P_t are equal (or nearly equal) to the expected values for which they are substituted. If this condition is not met, the estimate suffers from misspecification.[16] This flaw is likely to produce a

[15]Data on the gross domestic product (GDP) are released quarterly. The consumer price index, published monthly, is a better measure of Swiss inflation than the GDP deflator. The nominal GDP, and hence the deflator, are subject to larger revisions than the deflated GDP. The R^2s of the estimates decrease substantially when the consumer price index is replaced by the GDP deflator. Although the estimated coefficients remain virtually unchanged when the GDP is deflated with the consumer price index, the R^2s of the estimates decrease. Therefore, the officially deflated series for the GDP is used in this study. The interest rate for Euro-deposits in Swiss francs is considered the best indicator for the return on money market instruments in Switzerland. Published domestic rates are applicable to small investors; large investors are able to get Euromarket rates (about half a percentage point more than the domestic rate) even if they deposit their funds with a domestic bank.

[16]This can be seen with the terminology used in the appendix: while the method of free parameters searches for γ values that minimize the residual sum of squares, the method of determined parameters imposes arbitrary γ values.

Chart 1
Pascal Lag Estimates of Lag Weights for the Price Level Adjustment

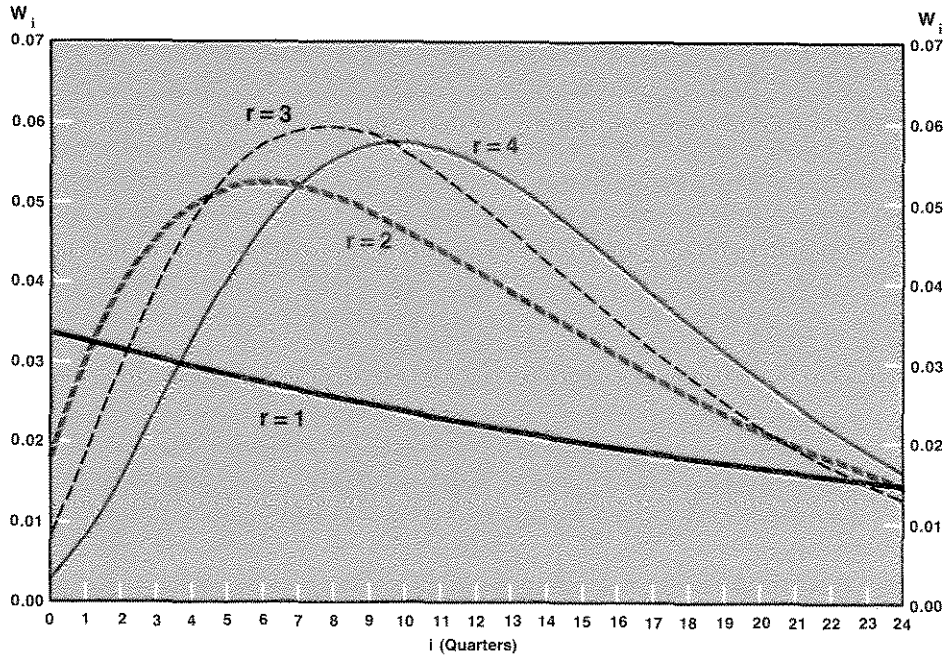


Chart 2
Pascal Lag Estimates of the Adjustment Path of the Price Level

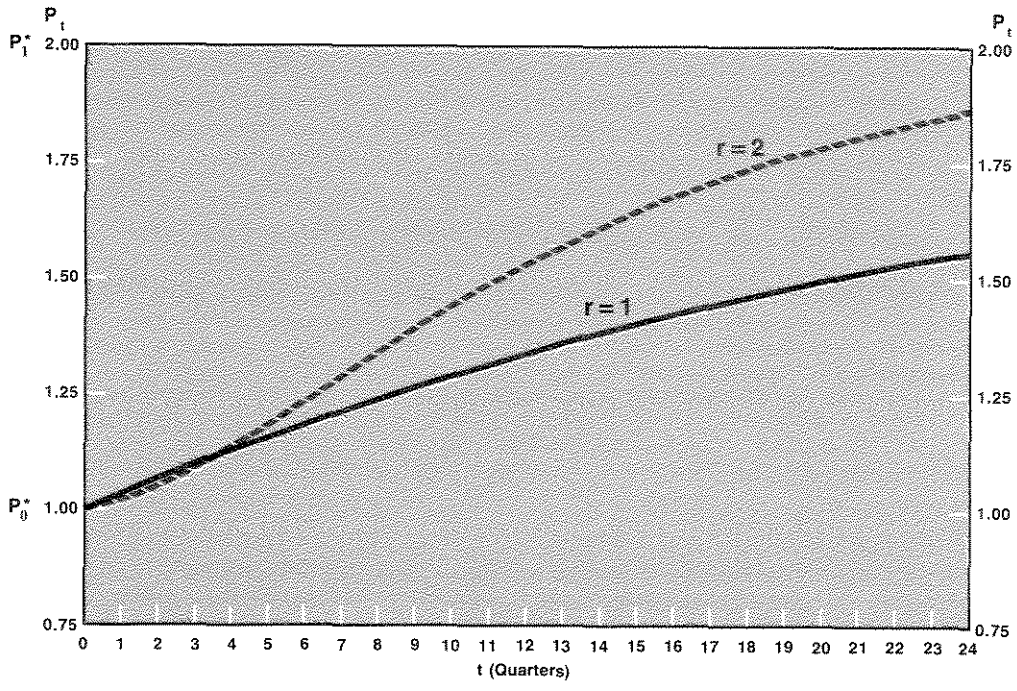


Table 2

Estimates of Pascal Lag (r = 2) Using the Method of Free Parameters: I/1973–IV/1986

λ	0.868* (53.17)
α_1	0.393 (0.97)
α_2	−0.042* (5.45)
$\bar{R}^2$	0.45
DW	2.05
SSR	0.0023

Absolute values of t-statistic in parentheses.
* Indicates statistical significance at the 5 percent level.

Table 3

Estimates of Pascal Lag (r = 2) Using the Method of Determined Parameters: I/1973 – IV/1986

λ	0.830* (56.93)
α_1	0.830* (2.45)
α_2	−0.045* (7.50)
$\bar{R}^2$	0.41
DW	1.65
SSR	0.0029

Absolute values of t-statistic in parentheses.
* Indicates statistical significance at the 5 percent level.

Table 4

Estimates of Pascal Lag (r = 2) Using the Method of Determined Parameters

	Starting Point of Sample Period							
	73.2	73.3	73.4	74.1	74.2	74.3	74.4	75.1
λ	0.854* (51.19)	0.628* (23.02)	0.715* (14.23)	0.758* (20.60)	0.772* (51.92)	0.803* (53.37)	0.780* (39.30)	0.892* (76.05)
α_1	0.628 (1.64)	0.586 (1.54)	1.198* (2.75)	1.553* (4.05)	0.482 (1.44)	0.799* (2.36)	0.175 (0.44)	0.869* (2.21)
α_2	−0.043* (6.15)	−0.023* (4.77)	−0.023* (3.50)	−0.031* (5.20)	−0.044* (8.56)	−0.050* (8.56)	−0.022* (3.01)	−0.056* (5.38)
$\bar{R}^2$	0.42	0.18	0.00	0.05	0.30	0.31	0.08	0.31
DW	1.79	0.66*	0.51*	0.65*	0.96*	1.08*	0.79*	1.75
SSR	0.0027	0.0088	0.0072	0.0050	0.0031	0.0027	0.0035	0.0014

Absolute values of t-statistic in parentheses.
* Indicates statistical significance at the 5 percent level.

poor fit and significant autocorrelation of the residuals. To get a feel for the magnitude of this problem, the model is reestimated with eight new starting points ranging from 1973.2 to 1975.1. Table 4 shows the outcome. The adjusted coefficient of determination ($\bar{R}^2$) varies widely with the starting value of the estimate. Six estimates show significant autocorrelation of the residuals. Only two estimates pass a Durbin-Watson test for misspecification; these two, with starting points 1973.2 and 1975.1, have the highest $\bar{R}^2$s in this series of estimates. Finally, the parameter estimates in these two cases are in line with those in tables 2 and 3.

Thus, while both methods of applying the Pascal lag to a relatively small data sample have their limitations, their estimates of the parameters of long-run money demand, as well as the dynamic adjustment process, are consistent.

SUMMARY AND CONCLUSIONS

This article deals with the estimation of money demand in Switzerland. During the period of monetary control since 1973, the public has adjusted real money balances to its desired level by means of price level changes. Thus, the estimation of money demand is tantamount to statistically tracking variations in the price level.

The Pascal lag technique frees the adjustment dynamics from the rigid corset of the partial adjustment process so frequently used in money demand studies. The statistical findings show that the partial adjustment process does not accurately describe how the price level adjusts to changes in its determinants. It takes approximately one and a half years for the adjustment speed of the price level to reach its maximum and about one more year before half of the necessary adjustment is completed.

Among the estimates that pass a test of misspecification, no income coefficient of money demand is statistically significantly different from one. Nevertheless, all point estimates are less than one. This result suggests that price level stability in Switzerland is more likely to be achieved with an M1 growth somewhat less than real income growth.

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Appendix
Implicit Time Trends in the Pascal Lag Estimation Using the Method of Free Parameters

This appendix demonstrates that using the method of free parameters to apply the Pascal lag to a finite sample size implies the estimation of time trends. The Pascal lag of order 2 serves as an example. The initial equation for this case is

$$\Delta p_t = (1-\lambda)^2 \sum_{i=0}^{\infty} (i+1)\lambda^i (\Delta m_{t-i} - \alpha_1 \Delta y_{t-i} - \alpha_2 \Delta R_{t-i}) + \varepsilon_t.$$

The infinite part of the equation is omitted by rewriting this equation as

$$\Delta p_t = (1-\lambda)^2 \sum_{i=0}^{t-2} (i+1)\lambda^i (\Delta m_{t-i} - \alpha_1 \Delta y_{t-i} - \alpha_2 \Delta R_{t-i}) - (t-1)\lambda^t E(\Delta p_0) + t\lambda^{t-1} E(\Delta p_1) + \varepsilon_t.$$

Using the method of free parameters means estimating this equation in the form:

$$\Delta p_t = (1-\lambda)^2 \sum_{i=0}^{t-2} (i+1)\lambda^i (\Delta m_{t-i} - \alpha_1 \Delta y_{t-i} - \alpha_2 \Delta R_{t-i}) - (t-1)\lambda^t \alpha_3 + t\lambda^{t-1} \alpha_4 + \varepsilon_t.$$

The estimation procedure is not designed to generate values of $\hat{\alpha}_3$ and $\hat{\alpha}_4$ that are equal to the corresponding $E(\Delta p)$ values. Instead, the maximum likelihood procedure will find values of these "free parameters" that minimize the sum of squared residuals:

$$\hat{\alpha}_3 = E(\Delta p_0) + \hat{\gamma}_1.$$

$$\hat{\alpha}_4 = E(\Delta p_1) + \hat{\gamma}_2.$$

Hence, implicitly, the method of free parameters respecifies the initial equation to

$$\Delta p_t = (1-\lambda)^2 \sum_{i=0}^{\infty} (i+1)\lambda^i (\Delta m_{t-i} - \alpha_1 \Delta y_{t-i} - \alpha_2 \Delta R_{t-i}) + \gamma_1 (1-t)\lambda^t + \gamma_2 t\lambda^{t-1} + \varepsilon_t.$$

The term thus added to the basic equation, $\gamma_1 (1-t)\lambda^t + \gamma_2 t\lambda^{t-1}$, is a time trend. The form of this time trend is limited: after a positive or negative value at the beginning of the sample, it eventually goes toward zero. The trend parameters, $\hat{\gamma}_1$ and $\hat{\gamma}_2$, however, cannot be estimated explicitly.