Appendix A: Determining the equilibria

Throughout, we will impose the boundary conditions $[(1-\theta_b)x - V^0 - \rho] < 0$ and $[(1-\theta_g)(1-\theta_b)x - (1-\theta_g)V^0 - (1-\theta_b)\rho] > 0$ for $g = h, l$. The first condition also ensures that lenders' offers to screen out uncreditworthy borrowers do not have the undesirable property that $C > R$. The second condition ensures that uncreditworthy types do not find the lenders' competitive offers to creditworthy types unattractive.

Lemma 1 *For borrowers of type-b, the Lender-I denies credit. For borrowers of types $g = h, l$ the Lender-I offers a contract from the set $Z_g^I(\rho^I) = \{(R_g^I, 0) : R_g^I \in [\underline{R}_g^I(\rho^I), \bar{R}_g]\}$, where $\underline{R}_g^I(\rho^I) = \frac{\rho^I}{1-\theta_g}$ and $\bar{R}_g = x - \frac{V^0}{1-\theta_g}$, $g = h, l$ are the first-best (zero-collateral) minimum and maximum repayments, respectively.*

Proof. Since $\pi_b^I < 0$, the Lender-I denies credit to the b-type. Since Lender-I knows borrower type, we can focus our attention without loss of generality to either creditworthy type $g = h, l$. We first show that Lender-I will always offer zero-collateral contracts.

We prove by contradiction. Suppose not, that is, there exists an offer with a positive collateral requirement from Lender-U, (R_g^1, C_g^1), that yields non-negative profits, $\pi_g^I(R_g^1, C_g^1) \geq 0$. Consider offer (R_g^2, C_g^2) with $R_g^2 > R_g^1$, $C_g^2 < C_g^1$ such that $V_g(R_g^1, C_g^1) = V_g(R_g^2, C_g^2)$, $g = h, l$. From $V_g(R_g^1, C_g^1) = V_g(R_g^2, C_g^2)$, it follows that $(1-\theta_g)(R_g^2 - R_g^1) = \theta_g(C_g^1 - C_g^2)$. Therefore, $\pi_g(R_g^2, C_g^2) - \pi_g(R_g^1, C_g^1) = \theta_g(1-\beta)(C_g^1 - C_g^2) > 0$. As long as $C_g^1 > 0$, Lender-I can reduce the collateral requirement to C_g^2, with offer (R_g^2, C_g^2) which provides borrower g with the same payoff as (R_g^1, C_g^1). All such deviations yield higher profits for Lender-I. Therefore, Lender-I will always choose a contract that sets its collateral requirement to zero.

The first-best (zero-collateral) minimum repayment is obtained by setting $\pi_g^I = 0$, $g = h, l$. The first-best (zero-collateral) maximum is obtained by setting $V_g(R_g^I, 0) = V^0, g = h, l$. ■

TABLE 1. Lender-U's offers under different candidate equilibria

Candidate equilibria	*Profit*	*Customer types accepting Lender-U's offer*	*Menu of contracts offered by Lender-U*	*Breakeven cutoff*
Screen-1	Π_S^1	$(h);(l)$	$(R_h^U, C_h^U);(R_l^U, C_l^U)$	$\tilde{\rho}_S^{b,h}$, $\tilde{\rho}_S^{h,l}$
Screen-2	Π_S^2	(h)	$(R_h^U, C_h^U);(R_l^0, C_l^0)$	$\tilde{\rho}_S^{b,h}$
Screen-3	Π_S^3	(l)	(R_l^U, C_l^U)	$\tilde{\rho}_S^{h,l}$
Hybrid-1	Π_Y^1	$(b,h);(l)$	$(\underline{R}_h^I, 0);(R_l^U, C_l^U)$	$\tilde{\rho}_P^1(\nu_h, \nu_l)$, $\tilde{\rho}_S^{h,l}$
Hybrid-2	Π_Y^2	(h,l)	(R_g^U, C_g^U)	$\tilde{\rho}_Y(\nu_h, \nu_l)$
Pool-1	Π_P^1	(b,h,l)	$(\underline{R}_l^I, 0)$	$\tilde{\rho}_P^1(\nu_h, \nu_l)$
Pool-2	Π_P^2	(b,h)	$(\underline{R}_h^I, 0)$	$\tilde{\rho}_P^2(\nu_h, \nu_l)$

Three categories of equilibria are characterized in terms of Lender-U's offers. For example, any equilibria wherein Lender-U's offers successfully screen borrower types are characterized as *separating equilibria* of the model.[1] A second category involves *pooling equilibria* wherein Lender-U's offer of a single contract is accepted by two or more borrower types. Finally, there is a third category of equilibria wherein Lender-U's offers involve the bunching (or pooling) of adjacent borrower types while screening the non-adjacent borrower type. This occurs, when, for example, Lender-U bunches the creditworthy borrowers (the l-type and the h-type) while screening the uncreditworthy borrower (b-type). Since Lender-U's equilibrium offer involves both pooling and screening, we characterize this third category of equilibria as *hybrid equilibria*. It is important to mention here that these are candidate equilibria which emerge as the final equilibria of the model for different values of the model parameters.

The three categories of candidate equilibria: pooling, screening, and hybrid are summarized in Table 1. Within each category, candidate-1 has a larger number of borrower types accepting offers from Lender-U than candidate-2 or candidate-3. For example, in candidate equilibrium *Hybrid-2*, Lender-U screens out the b-type, but in *Hybrid-1* it pools them with h-types. If Lender-U can screen the b-type from the h-type, but not sort between the h-type and the l-type, then its offers in *Screen-2* would be accepted by the h-types. Conversely, if Lender-U can sort creditworthy types, but not screen out the b-type (or even pool them with h-types), Lender-U's offer in *Screen-3* would be accepted by the l-types. However, if Lender-U can sort between all borrower types, it can offer *Screen-1* whose profits dominate those of *Screen-2* and *Screen-3*. Similarly, for a given distribution of borrower types, the Lender-U's offers in *Hybrid-1* dominate those in *Pool-2*. There is no equilibrium in which Lender-U bunches the non-adjacent, b- and l-types.

Lender-U's offer in equilibria

Lender-U's optimization problem can be written as follows:[2]

$$\max \quad \Pi^U \equiv \nu_b \pi_b^U + \nu_h \pi_h^U + \nu_l \pi_l^U, \tag{1}$$

[1] The terms "sorting", "screening" and "separating" are used interchangeably. Also, the terms "bunching" and "pooling" are used interchangeably. A successful sorting occurs when, conditional on each borrower type accepting Lender-U's offers, Lender-U makes non-negative profits overall. Unless otherwise mentioned, we will restrict attention to successful sorting (and pooling) in what follows.

[2] In this section, for the most part, we drop the superscript denoting offers by borrower U. Therefore, unless otherwise mentioned, we are considering profits and offers of Lender-U only. We will re-introduce superscripts below.

where $\pi_k^U = (1-\theta_k)R_k^U + \beta\theta_k C_k^U - \rho^U$, $k = b, h, l$ subject to the following participation constraints

$$\begin{aligned} V_b(R_b, C_b) &\leq V^0 & (2) \\ V_h(R_h, C_h) &\geq \bar{V}_h^I & (3) \\ V_l(R_l, C_l) &\geq \bar{V}_l^I & (4) \end{aligned}$$

and the following incentive compatibility constraints

$$\begin{aligned} V_b(R_b, C_b) &\geq V_b(R_h, C_h) & (5) \\ V_b(R_b, C_b) &\geq V_b(R_l, C_l) & (6) \\ V_h(R_h, C_h) &\geq V_h(R_b, C_b) & (7) \\ V_h(R_h, C_h) &\geq V_h(R_l, C_l) & (8) \\ V_l(R_l, C_l) &\geq V_l(R_b, C_b) & (9) \\ V_l(R_l, C_l) &\geq V_l(R_h, C_h). & (10) \end{aligned}$$

Note that since $\pi_b^U < 0$, Lender-U does not offer contract (R_b, C_b) in equilibrium. Therefore, (2), (7) and (9) are redundant. Moreover, one may replace $V_b(R_b, C_b)$ with the b-type's reservation utility V^0 on the left-hand side of (5) and(6). Consequently, the above maximization problem in (1)-(10) reduces to

$$\max \quad \Pi^U \equiv \nu_b \pi_b^U + \nu_h \pi_h^U + \nu_l \pi_l^U, \tag{11}$$

where $\pi_k^U = (1-\theta_k)R_k^U + \beta\theta_k C_k^U - \rho^U$, $k = b, h, l$ subject to the following participation constraints

$$\begin{aligned} V_h(R_h, C_h) &\geq \bar{V}_h^I & (12) \\ V_l(R_l, C_l) &\geq \bar{V}_l^I & (13) \end{aligned}$$

and the following incentive compatibility constraints

$$\begin{aligned} V^0 &\geq V_b(R_h, C_h) & (14) \\ V^0 &\geq V_b(R_l, C_l) & (15) \\ V_h(R_h, C_h) &\geq V_h(R_l, C_l) & (16) \\ V_l(R_l, C_l) &\geq V_l(R_h, C_h). & (17) \end{aligned}$$

Lemma 2 *In any equilibirum offer by Lender-U, its expected profits from loans to l-types are non-negative, $\pi_l^U(R_l, C_l) \geq 0$.*

Proof. Suppose not, that is there exist equilibria in which $\pi_l^U(R_l, C_l) < 0$. It follows that $\pi_b^U(R_l, C_l) < \pi_h^U(R_l, C_l) < \pi_l^U(R_l, C_l) < 0$. Lender-$U$ can always drop this contract and increase profits. Therefore, in any equilibrium, it is always the case that $\pi_l^U(R_l, C_l) \geq 0$. ■

Lemma 3 *In any equilibirum offer by Lender-U that is accepted by the l-type, the IR constraint of the l-type, (13), must bind.*

Proof. *Case 1: Lender-U does not sort h-type from l-type*

We prove by contradiction. Suppose not, that is, there exists a solution to (11)-(15)[3] characterized by (R_g^1, C_g^1) such that

$$V_l(R_g^1, C_g^1) > \bar{V}_l^I \tag{18}$$

We will show that there exists an offer (R_g^2, C_g^2) such that it satisfies (13) but yields higher profit. We begin by characterizing this contract. Consider contract (R_g^2, C_g^2), where $R_g^2 > R_g^1$, $C_g^2 < C_g^1$ such that

$$V_b(R_g^1, C_g^1) = V_b(R_g^2, C_g^2) \tag{19}$$

It follows that

$$\begin{aligned} V_h(R_g^2, C_g^2) &< V_h(R_g^1, C_g^1) \\ V_l(R_g^2, C_g^2) &< V_l(R_g^1, C_g^1) \end{aligned} \tag{20}$$

Following the last inequality, we focus our attention to (R_g^2, C_g^2) such that

$$V_l(R_g^1, C_g^1) > V_l(R_g^2, C_g^2) \geq \bar{V}_l^I \tag{21}$$

Since (R_g^1, C_g^1) is a solution, it satisfies all the constraints. Consequently, we can show that offer (R_g^2, C_g^2) in (19) and (21) satisfies all other constraints as well. Constraints (13), (14) and (15) are satisfied by construction.[4]

It remains to be shown that (12) is satisfied. We prove by contradiction. Suppose not, then $V_h(R_g^1, C_g^1) \geq \bar{V}_h^I > V_h(R_g^2, C_g^2)$. It follows that $\rho^I < (1-\theta_h)R_g^2 + \theta_h C_g^2$. From (21), we obtain $\rho^I \geq (1-\theta_l)R_g^2 + \theta_l C_g^2$. Combining the two, we have $(1-\theta_h)R_g^2 + \theta_h C_g^2 > (1-\theta_l)R_g^2 + \theta_l C_g^2$ or

$$(\theta_h - \theta_l)(C_g^2 - R_g^2) > 0$$

Since $\theta_h > \theta_l$, it must be the case that $C_g^2 > R_g^2$. This is impossible given our intial assumption $[(1-\theta_b)x - V^0 - \rho^I] < 0$, so that all lender offers to creditworthy types have the property $R \geq C$. We have a contradiction. It must be the case that (12) is satisfied.

[3] Since Lender-U does not sort, h-type and l-types are bunched with a single offer. Consequently, (16) and (17) are trivially satisfied.

[4] Since Lender-U has only one offer for all creditworthy types, (14) and (15) are the same constraint.

Finally, since $R_g^2 > R_g^1$, $C_g^2 < C_g^1$, we have using (19) that $\pi_h(R_g^2, C_g^2) > \pi_h(R_g^1, C_g^1)$. Also, $\pi_l(R_g^2, C_g^2) > \pi_l(R_g^1, C_g^1)$. It follows that $\Pi(R_g^2, C_g^2) > \Pi(R_g^1, C_g^1)$. Therefore, (R_g^1, C_g^1) cannot be an equilibrium because Lender-U can offer an alternative contract of the form (R_g^2, C_g^2) and increase profits. Such profitable deviations are not possible only if $V_l(R_g, C_g) = \bar{V}_l^I$. Therefore (13) must bind in equilibrium.

Case 2: when Lender-U sorts creditworthy types.

We prove by contradiction. Suppose not, that is, there exists a solution to (11)-(17) characterized by $\{(R_h, C_h), (R_l^1, C_l^1)\}$ such that

$$V_l(R_l^1, C_l^1) > \bar{V}_l^I \tag{22}$$

We will show that there exist a menu $\{(R_h, C_h), (R_l^2, C_l^2)\}$ such that it satisfies (13) but yields higher profit. We begin by characterizing this contract. Consider contract (R_l^2, C_l^2), where $R_l^2 > R_l^1$, $C_l^2 < C_l^1$ such that

$$V_b(R_l^1, C_l^1) = V_b(R_l^2, C_l^2) \tag{23}$$

This implies

$$\begin{aligned} V_h(R_l^2, C_l^2) &< V_h(R_l^1, C_l^1) \\ V_l(R_l^2, C_l^2) &< V_l(R_l^1, C_l^1) \end{aligned} \tag{24}$$

Following this, we can choose (R_l^2, C_l^2) such that either of the following are true

$$V_l(R_l^1, C_l^1) > V_l(R_l^2, C_l^2) \geq V_l(R_h, C_h) \geq \bar{V}_l^I \tag{25}$$

$$V_l(R_l^1, C_l^1) > V_l(R_l^2, C_l^2) \geq \bar{V}_l^I > V_l(R_h, C_h) \tag{26}$$

First, we show that the menu $\{(R_h, C_h), (R_l^2, C_l^2)\}$ in (23) and either (25) or (26) satisfies all other constraints as well. Constraints (12) and (14) are trivially satisfied. Constraint (13), (15) and (17) are satisfied by construction. Lastly, (16) is satisfied because $V_h(R_h, C_h) \geq V_l(R_l^1, C_l^1) > V_l(R_l^2, C_l^2)$.

Next, we show that by offering (R_l^2, C_l^2) instead of (R_l^1, C_l^1), Lender-U increases profits. Since $R_l^2 > R_l^1$, $C_l^2 < C_l^1$, we have using (23)[5],

$$\pi_l(R_l^2, C_l^2) > \pi_l(R_l^1, C_l^1).$$

Therefore, menu $\{(R_h, C_h), (R_l^1, C_l^1)\}$ cannot be an equilibrium because Lender-U can offer an alternative contract of the form $\{(R_h, C_h), (R_l^2, C_l^2)\}$ and increase profits.

[5] This follows from $\pi_l(R_l^2, C_l^2) - \pi_l(R_l^1, C_l^1) = \frac{\theta_b(1-\theta_l)-\beta\theta_l(1-\theta_b)}{(1-\theta_b)}(C_l^1 - C_l^2)$ and $\theta_b(1-\theta_l) - \beta\theta_l(1-\theta_b) > (1-\beta)\theta_l(1-\theta_b) > 0$.

Note that this holds for either (25) or (26). For (25), all deviations from (R_l^1, C_l^1) to (R_l^2, C_l^2) yield higher profits, unless $V_l(R_l, C_l) = V_l(R_h, C_h)$; in which case, Lender-$U$ bunches h-types and l-types and we follow the proof as given in Case 1 above. Alternatively, for (26) such profitable deviations are not possible only if $V_l(R_l, C_l) = \bar{V}_l^I$. Therefore (13) must bind in equilibrium. ■

Lemma 4 *In any equilibrium wherein Lender-U screens out the b-type, the IC constraint of the b-type w.r.t the h-type, (14), must bind.*

Proof. *Case 1: when Lender-U sorts creditworthy types.*

We prove by contradiction. Suppose not, that is, there exists a solution to (11)-(17) characterized by $\{(R_h^1, C_h^1), (R_l, C_l)\}$ such that

$$V^0 > V_b(R_h^1, C_h^1)$$

We will show that there exist menus such as $\{(R_h^2, C_h^2), (R_l, C_l)\}$ that satisfy (14) all the other constraints but yield higher profits. We begin by characterizing such contracts. Consider contract (R_h^2, C_h^2), where $R_h^2 > R_h^1$, $C_h^2 < C_h^1$ such that

$$V_h(R_h^1, C_h^1) = V_h(R_h^2, C_h^2). \tag{27}$$

It follows that

$$\begin{aligned} V_l(R_h^2, C_h^2) &< V_l(R_h^1, C_h^1) \\ V_b(R_h^2, C_h^2) &> V_b(R_h^1, C_h^1) \end{aligned} \tag{28}$$

Therefore, if $V^0 > V_b(R_h^1, C_h^1)$, we can choose (R_h^2, C_h^2) so that

$$V^0 \geq V_b(R_h^2, C_h^2) > V_b(R_h^1, C_h^1).$$

Since $\{(R_h^1, C_h^1), (R_l, C_l)\}$ is a solution, it satisfies all the constraints. Consequently, we can show that menus $\{(R_h^2, C_h^2), (R_l, C_l)\}$ satisfy all the constraints as well. Constraints (13) and (15) are trivially satisfied. Constraint (12) and (16) are satisfied by construction. Lastly, (17) is satisfied because $V_l(R_l, C_l) \geq V_l(R_h^1, C_h^1) > V_l(R_h^2, C_h^2)$.

But, since $R_h^2 > R_h^1$ and $C_h^2 < C_h^1$, we use (27) to obtain

$$\pi_h(R_h^2, C_h^2) > \pi_h(R_h^1, C_h^1)$$

Therefore, $\{(R_h^1, C_h^1), (R_l, C_l)\}$ cannot be an equilibrium because Lender-U can offer an alternative contract of the form $\{(R_h^2, C_h^2), (R_l, C_l)\}$ and increase profits. Such profitable deviations

are not possible only if $V^0 = V_b(R_h, C_h)$. Therefore (14) must bind in equilibrium.
Case 2: Lender-U does not sort h-type from l-type

This holds for either creditworthy type, yielding two sets of equilibria where Lender-U screens out just the b-type. The first occurs when $g = h$, and Lender-U captures only the h-type by screening them from the b-type, as described in the candidate equilibria *Screen-2*. The second occurs when $g = l$ and Lender-U captures both h- and l-types by bunching them and screening them from the b-type, as described in the candidate equilibria *Hybrid-2*.

We prove by contradiction for *Hybrid-2*. Suppose not, that is, there exists a solution to (11)-(15)[6] characterized by (R_g^1, C_g^1), where (R_g^1, C_g^1) is the offer to both the h-type and the l-type, and $V^0 > V_b(R_g^1, C_g^1)$. Note that one can find an alternative contract (R_g^2, C_g^2) with $R_g^2 > R_g^1$ and $C_g^2 < C_g^1$, such that

$$V_l(R_g^1, C_g^1) = V_l(R_g^2, C_g^2). \tag{29}$$

It follows that

$$\begin{aligned} V_h(R_g^2, C_g^2) &> V_h(R_g^1, C_g^1) \\ \text{and } V_b(R_g^2, C_g^2) &> V_b(R_g^1, C_g^1). \end{aligned} \tag{30}$$

Following this, we can restrict our attention to contracts such that

$$V^0 \geq V_b(R_g^2, C_g^2) > V_b(R_g^1, C_g^1). \tag{31}$$

Since (R_g^1, C_g^1) is a solution, it satisfies all the constraints. Consequently, we can show that offer (R_g^2, C_g^2) in (29) and (31) satisfies all other constraints as well. Constraints (13), (14) and (15) are satisfied by construction.[7] Lastly, (12) is satisfied from (30).

In offering (R_g^2, C_g^2) instead of (R_g^1, C_g^1), Lender-U's change in profits from the l-type and h-type are given by

$$\begin{aligned} \Delta\pi_l^U &= (1-\beta)\theta_l(C_g^1 - C_g^2) > 0 \\ \Delta\pi_h^U &= \frac{\theta_l(1-\theta_h) - \beta\theta_h(1-\theta_l)}{(1-\theta_l)}(C_g^1 - C_g^2) \end{aligned}$$

Since bunching is only feasible for $\nu_l \geq \nu_h$ and

$$(1-\beta)\theta_l > \frac{(1-\beta)\theta_l(1-\theta_h)}{(1-\theta_l)} > \frac{\theta_l(1-\theta_h) - \beta\theta_h(1-\theta_l)}{(1-\theta_l)}$$

[6] Since Lender-U does not sort, h-type and l-types are bunched with a single offer. Consequently, (16) and (17) are trivially satisfied.

[7] Since Lender-U has only one offer for all creditworthy types, (14) and (15) are the same constraint.

Therefore Lender-U's offer of (R_g^2, C_g^2) to both h-type and l-types yields higher profits than (R_g^1, C_g^1). That is, $\Pi(R_g^2, C_g^2) > \Pi(R_g^1, C_g^1)$. This implies that (R_g^1, C_g^1) cannot be an equilibrium. Such deviations are no longer possible if $V_b(R_g, C_g) = V^0$.

Proceeding exactly as above, we can prove the same for candidate equilibria *Screen-2*, where Lender-U's offers are accepted by the h-types only. ■

Lemma 5 *In any equilibrium wherein Lender-U sorts the h-types from the l-type, the IC constraint of the h-type w.r.t the l-type,(16), must bind.*

Proof. We prove by contradiction. Suppose not, that is, there exists a solution to (11)-(17) characterized by $\{(R_h, C_h), (R_l^1, C_l^1)\}$ such that

$$V_h(R_h, C_h) > V_h(R_l^1, C_l^1)$$

We will show that there exist a menus of contracts $\{(R_h, C_h), (R_l^2, C_l^2)\}$ such that it satisfies (16) and all the other constraints but yields higher profit. We begin by characterizing this contract. Consider contract (R_l^2, C_l^2), where $R_l^2 > R_l^1$, $C_l^2 < C_l^1$ such that

$$V_l(R_l^1, C_l^1) = V_l(R_l^2, C_l^2) = \bar{V}_l^I \tag{32}$$

It follows that

$$\begin{aligned} V_h(R_l^2, C_l^2) &> V_h(R_l^1, C_l^1) \\ V_b(R_l^2, C_l^2) &> V_b(R_l^1, C_l^1) \end{aligned}$$

Therefore, if $V_h(R_l^2, C_l^2) > V_h(R_l^1, C_l^1)$, we can choose (R_l^2, C_l^2) so that

$$V_h(R_h, C_h) \geq V_h(R_l^2, C_l^2) > V_h(R_l^1, C_l^1) \tag{33}$$

Since $\{(R_h, C_h), (R_l^1, C_l^1)\}$ is a solution, it satisfies all the constraints. Consequently, we can show that the menu $\{(R_h, C_h), (R_l^2, C_l^2)\}$ in (32) and (33) satisfies all the constraints as well. Constraints (12) and (14) are trivially satisfied. Constraint (13) and (17) are satisfied by construction.

For (15), we prove by contradiction. Suppose not, then $V_b(R_l^2, C_l^2) > V^0$. In addition, Lemma 4 implies $V_b(R_l^2, C_l^2) > V^0 = V_b(R_h, C_h)$. That is, $(1-\theta_b)(R_h - R_l^2) > \theta_b(C_l^2 - C_h)$. Also, because (33) holds, it follows that $\theta_h(C_l^2 - C_h) \geq (1-\theta_h)(R_h - R_l^2)$. Combining both,

$$\begin{aligned} (\frac{1-\theta_b}{\theta_b})(R_h - R_l^2) &> (C_l^2 - C_h) \geq (\frac{1-\theta_h}{\theta_h})(R_h - R_l^2) \\ (\frac{1-\theta_b}{\theta_b} - \frac{1-\theta_h}{\theta_h})(R_h - R_l^2) &> 0 \end{aligned}$$

Since the first expression is negative, this would imply $R_l^2 > R_h$. Similarly we can show that $C_l^2 < C_h$. However, since (17) is satisfied, we get $(1-\theta_l)(R_h - R_l^2) > \theta_l(C_l^2 - C_h)$. Again, using (33) it follows that $\theta_h(C_l^2 - C_h) \geq (1-\theta_h)(R_h - R_l^2)$. Combining both,

$$\begin{aligned}(\frac{1-\theta_l}{\theta_l})(R_h - R_l^2) &> (C_l^2 - C_h) \geq (\frac{1-\theta_h}{\theta_h})(R_h - R_l^2) \\ (\frac{1-\theta_l}{\theta_l} - \frac{1-\theta_h}{\theta_h})(R_h - R_l^2) &> 0\end{aligned}$$

Now, since the first expression is positive, this would imply $R_l^2 < R_h$. Similarly we can show that $C_l^2 > C_h$. We have a contradiction. Therefore, it cannot be the case that $V_b(R_l^2, C_l^2) > V^0$ and (15) is satisfied.

Since $R_l^2 > R_l^1$, $C_l^2 < C_l^1$, using (32), we get

$$\pi_l(R_l^2, C_l^2) > \pi_l(R_l^1, C_l^1)$$

Therefore, menu $\{(R_h, C_h), (R_l^1, C_l^1)\}$ cannot be an equilibrium because Lender-U can offer an alternative contract of the form $\{(R_h, C_h), (R_l^2, C_l^2)\}$ and increase profits. Such profitable deviations are not possible only if $V_h(R_h, C_h) = V_h(R_l, C_l)$. Therefore (16) must bind in equilibrium. ■

Lender-U's offers in screening equilibria

Consequently, the above maximization problem in (1)-(10) reduces to

$$\max \ \Pi^U \equiv \nu_h \pi_h^U + \nu_l \pi_l^U, \tag{34}$$

where $\pi_k^U = (1-\theta_k)R_k^U + \beta\theta_k C_k^U - \rho^U$, subject to the following participation constraints

$$\begin{aligned}V_h(R_h, C_h) &\geq \bar{V}_h^I & (35)\\ V_l(R_l, C_l) &= \bar{V}_l^I & (36)\end{aligned}$$

and the following incentive compatibility constraints

$$\begin{aligned}V^0 &= V_b(R_h, C_h) & (37)\\ V^0 &\geq V_b(R_l, C_l) & (38)\\ V_h(R_h, C_h) &= V_h(R_l, C_l) & (39)\\ V_l(R_l, C_l) &\geq V_l(R_h, C_h). & (40)\end{aligned}$$

Using the equations (35)-(40), we obtain expressions for (R_h, C_h) and (R_l, C_l) as follows:

$$\begin{aligned}
R_g^U &\geq R_l \geq R_l^U, \\
R_h^U &\geq R_h \geq R_g^U \\
C_g^U &\leq C_l \leq C_l^U \\
C_h^U &\leq C_h \leq C_g^U
\end{aligned}$$

where

$$R_l^U = C_l^U = \rho^I, \tag{41}$$

$$R_g^U = \frac{\theta_b}{\theta_b - \theta_l}\rho^I - \frac{\theta_l}{\theta_b - \theta_l}[(1-\theta_b)x - V^0] \tag{42}$$

$$C_g^U = -\frac{1-\theta_b}{\theta_b - \theta_l}\rho^I + \frac{1-\theta_l}{\theta_b - \theta_l}[(1-\theta_b)x - V^0] \tag{43}$$

$$R_h^U = \frac{\theta_b}{\theta_b - \theta_h}\rho^I - \frac{\theta_h}{\theta_b - \theta_h}[(1-\theta_b)x - V^0] \tag{44}$$

$$C_h^U = -\frac{1-\theta_b}{\theta_b - \theta_h}\rho^I + \frac{1-\theta_h}{\theta_b - \theta_h}[(1-\theta_b)x - V^0]. \tag{45}$$

Since we assume $[(1-\theta_b)x - V^0 - \rho^I] < 0$ and $[(1-\theta_g)(1-\theta_b)x - (1-\theta_g)V^0 - (1-\theta_b)\rho^I] > 0$ for $g = h, l$ all of the offers given by (41)-(45) are strictly positive.

From (35)-(40), note that if the offer to h-types is $(R_h, C_h) = (R_h^U, C_h^U)$, then the offer to l-types is $(R_l, C_l) = (R_l^U, C_l^U)$ and vice-versa. Likewise, $(R_h, C_h) = (R_l, C_l) = (R_g^U, C_g^U)$ is an offer satisfying (35)-(40) in which Lender-U bunches both h-types and l-types.

Lemma 6 *In any equilibrium wherein Lender-U sorts the h-types from the l-type, the IR constraint of the l-type, (35) must bind.*

Proof. First note that $V_h(R_h^U, C_h^U) = \bar{V}_h^I$. Also lemmas 3-5 hold.

We prove by contradiction. Suppose not. That is, there exists a menu $\{(R_h^2, C_h^2), (R_l^2, C_l^2)\}$ which satifies (35)-(40) where $R_h^2 < R_h^U$ and $C_h^2 > C_h^U$ such that $V_b(R_h^2, C_h^2) = V_b(R_h^U, C_h^U)$. It follows that

$$V_h(R_h^2, C_h^2) > V_h(R_h^U, C_h^U) = \bar{V}_h^I$$

or (35) is slack. Also, from (36) and (39) it follows that $R_l^2 > R_l^U$, $C_l^2 < C_l^U$. Therefore, replacing $\{(R_h^U, C_h^U), (R_l^U, C_l^U)\}$ by $\{(R_h^2, C_h^2), (R_l^2, C_l^2)\}$ would increase profits from the l-types but decrease profits from the h-types as follows:

$$\begin{aligned}
\Delta\pi_l^U &= (1-\beta)\theta_l \frac{(1-\theta_l)}{(1-\theta_b)}\left(\frac{\theta_b - \theta_h}{\theta_h - \theta_l}\right)(C_h^2 - C_h^U) > 0 \\
\Delta\pi_h^U &= \frac{\theta_b(1-\theta_h) - \beta\theta_h(1-\theta_b)}{(1-\theta_b)}(C_h^U - C_h^2) < 0
\end{aligned}$$

The effect on total profits Π^U depends on ν_h and ν_l as long as each are strictly positive. However, if the total effect is positive, all deviations with menus of the type $\{(R_h^2, C_h^2), (R_l^2, C_l^2)\}$ yield higher profits than $\{(R_h^U, C_h^U), (R_l^U, C_l^U)\}$.[8] That is, profits are maximized by offering (R_g^U, C_g^U) which does not induce separation. Therefore, in an equilibrium that induces separation of the h-types and l-types, (35) must bind. ■

We are now in a position to list the candidate equilibria of the model. These are given below in Propositions 7-14. They are candidate equilibria of the model. In what follows, we hold the value of Lender-U's cost of funds fixed at ρ^U and vary the value of Lender-I's cost of funds, ρ^I. Depending on the distribution of borrower types ν_b and ν_h, and the value of ρ^I we can determine which of the following candidate equilibria will emerge as the final equilibrium of the model. Hereafter, we reintroduce the superscripts I and U for lenders' offers and profits.

Candidate equilibrium: Screen-1

We can now state Lender-U's offers under *Screen-1* in terms of the following proposition.

Proposition 7 *A pure strategy equilibrium wherein Lender-U separates all borrower types occurs when* $\rho^I \geq \max(\tilde{\rho}_S^{b,h}, \tilde{\rho}_S^{h,l})$ *and is characterized as follows:*
(a) Lender-U offers menu $\{(R_h^U, C_h^U), (R_l^U, C_l^U)\}$ *given by (41), (44) and (45).*
(b) Lender-I offers $(\underline{R}_h^I, 0)$ *to h-types and* $(\underline{R}_l^I, 0)$ *to l-types.*
(c) b-types types reject offers from either lender. Both creditworthy types accept offers from Lender-U, and

$$\tilde{\rho}_S^{h,l} = \frac{1}{1-(1-\beta)\theta_l}\rho^U \tag{46}$$
$$\tilde{\rho}_S^{b,h} = \frac{\theta_b - \theta_h}{\theta_b(1-\theta_h) - \beta\theta_h(1-\theta_b)}\rho^U + \frac{(1-\beta)\theta_h(1-\theta_h)}{\theta_b(1-\theta_h) - \beta\theta_h(1-\theta_b)}[(1-\theta_b)x - V^0]. \tag{47}$$

Proof. In any screening equilibrium wherein Lender-U separates all borrower types, Lemmas 1-6 must hold. This implies that Lender-U offers menu $\{(R_h^U, C_h^U), (R_l^U, C_l^U)\}$. Moreover, in equilibrium, Lender-I gives the break-even contract offers to each type, that is $(\underline{R}_h^I, 0)$ to h-types and $(\underline{R}_l^I, 0)$ to l-types, which the borrowers reject. Finally, for Lender-U to screen the l-type from the h-type, profits $\pi_l^U(R_l^U, C_l^U) \geq 0$. This occurs when $\rho^I \geq \tilde{\rho}_S^{h,l} = \frac{1}{1-(1-\beta)\theta_l}\rho^U$. Similarly, Lender-$U$ can screen the h-type from the b-type, if $\pi_h^U(R_h^U, C_h^U) \geq 0$. That is if $\rho^I \geq \tilde{\rho}_S^{b,h}$, where

$$\tilde{\rho}_S^{b,h} = \frac{\theta_b - \theta_h}{\theta_b(1-\theta_h) - \beta\theta_h(1-\theta_b)}\rho^U + \frac{(1-\beta)\theta_h(1-\theta_h)}{\theta_b(1-\theta_h) - \beta\theta_h(1-\theta_b)}[(1-\theta_b)x - V^0].$$

[8] Strictly speaking, if $\Delta\Pi^U \equiv \nu_h\Delta\pi_l^U + \nu_l\Delta\pi_l^U = 0$, then all points satisfying (35)-(40) can be supported as equilbira. However, we restrict our attention to the screening offer given in lemma 6.

■

In order to screen all borrower types, we must have $\rho^I > \max(\tilde{\rho}_S^{b,h}, \tilde{\rho}_S^{h,l})$. However, if $\tilde{\rho}_S^{h,l} > \rho^I \geq \tilde{\rho}_S^{b,h}$ or $\tilde{\rho}_S^{b,h} > \rho^I \geq \tilde{\rho}_S^{h,l}$, Lender-$U$ still has the option to just screen one creditworthy type, as given by the following propositions.

Candidate Equilibrium: Screen-2

Proposition 8 *A pure strategy equilibrium wherein Lender-U screens out just the b-type and lends to the h-types only, occurs when* $\tilde{\rho}_S^{h,l} > \rho^I \geq \tilde{\rho}_S^{b,h}$ *and is characterized as follows:*
(a) Lender-U offers menu $\{(R_h^U, C_h^U); (R_l^0, C_l^0)\}$, *with* (R_h^U, C_h^U) *given by (44) and (45) and* (R_l^0, C_l^0) *given as in (b) below*
(b) Lender-I offers $(\underline{R}_h^I, 0)$ *to h-types and* $(R_l^I, 0)$ *to l-types where* $\pi_l^U(R_l^0, C_l^0) = 0$, $V_h(\underline{R}_h^I, 0) = V_h(R_l^0, C_l^0)$, *and* $V_l(R_l^0, C_l^0) = V_l(R_l^I, 0)$.
(c) b-types reject offers from either lender. h-types accept offers from Lender-U, but l-types accept the offer from Lender-I.

Proof. The maximization problem for Screen-2 is the same as that of Screen-1, except for the fact that now $\tilde{\rho}_S^{h,l} > \rho^I \geq \tilde{\rho}_S^{b,h}$. Therefore, in Screen-2, all the results of Screen-1 hold except for the lenders' offers to the l-type. This implies that since $\tilde{\rho}_S^{h,l} > \rho^I \geq \tilde{\rho}_S^{b,h}$, (R_l^U, C_l^U) as given in (41), if offered, would yield negative profits for Lender-U. Also, so that h-types do not find the offer to the l-types attractive, any such offer would have to satisfy (39). Therefore, the best contract (highest borrower payoff) Lender-U can offer to the l-type is given by (R_l^0, C_l^0), where $\pi_l^U(R_l^0, C_l^0) = 0$. For any such offer, (36) is not satisfied: $V_l(R_l^0, C_l^0) < \bar{V}_l^I$. This implies that Lender-I does not need to offer $(\underline{R}_l^I, 0)$ to retain l-types. Lender-I can offer $(R_l^I, 0)$ instead, so that $V_l(R_l^0, C_l^0) = V_l(R_l^I, 0)$ and still retain the l-type with higher profits. ■

Candidate equilibrium: Screen-3

Proposition 9 *A pure strategy equilibrium wherein Lender-U separates only the l-types occurs when* $\tilde{\rho}_S^{b,h} > \rho^I \geq \tilde{\rho}_S^{h,l}$ *and* $\tilde{\rho}^h \geq \rho^I \geq \tilde{\rho}_S^{h,l}$, *where* $\tilde{\rho}^h = \frac{1}{1-(1-\beta)\theta_h}\rho^U$, *is characterized as follows:*
(a) Lender-U offers menu (R_l^U, C_l^U) *given by (41).*
(b) Lender-I offers $(\underline{R}_h^I, 0)$ *to h-types and* $(\underline{R}_l^I, 0)$ *to l-types,*
(c) Only the l-type accepts the offer from Lender-U. The h-type is retained by Lender-I.

Proof. The maximization problem for Screen-3 is the same as that of Screen-1, except for the fact that now $\tilde{\rho}_S^{b,h} > \rho^I \geq \tilde{\rho}_S^{h,l}$. Lender-U can simply make and offer to (R_l^U, C_l^U) as given in (41). This implies that since $\tilde{\rho}_S^{b,h} > \rho^I \geq \tilde{\rho}_S^{h,l}$, (R_h^U, C_h^U) as given in (44)-(45), if offered, would yield negative profits for Lender-U. Therefore, Lender-U cannot screen out the b-types. Lender-I continues to offer $(\underline{R}_h^I, 0)$ as before. Note that while Lender-U cannot match this offer, Lender-I cannot raise R_h^I because the h-types are just indifferent between its offer of $(\underline{R}_h^I, 0)$ and Lender-U's offer of (R_l^U, C_l^U). It is important, therefore, that Lender-U's profits from h-types are non-positive. That is $\tilde{\rho}^h \geq \rho^I$, where $\tilde{\rho}^h \equiv \frac{1}{1-(1-\beta)\theta_h}\rho^U$. ■

Equilibria with Lender-U pooling the b-type

When the proportion of the b-type is sufficiently small, Lender-U can choose to pool them with creditworthy types. There are two such types of equilibria; one, where b-types are pooled with the h-type and the other where all types are pooled together.

Lemma 10 *In any equibrium where the Lender-U pools the bad-risk type, it offers a zero collateral contract.*

Proof. The proof is by contradiction. Suppose not, that is, there exists a pooling equilibrium where Lender-U's offers (R_P^1, C_P^1) and pools all borrowers. Since l-types accept this contract, we must have from Lemma 3 that $V_l(R_P^1, C_P^1) = \bar{V}_l^I$. Consider alternative offer (R_P^2, C_P^2) with $R_P^2 > R_P^1$ and $C_P^2 < C_P^1$, such that $V_l(R_P^2, C_P^2) = V_l(R_P^1, C_P^1)$. It follows that

$$\begin{aligned} V_h(R_P^2, C_P^2) &> V_h(R_P^1, C_P^1) \\ V_b(R_P^2, C_P^2) &> V_b(R_P^1, C_P^1) \end{aligned}$$

Therefore, both b-types and h-types accept this new contract as it yields them higher payoff. But (R_P^2, C_P^2) yields Lender-U higher profits than (R_P^1, C_P^1).[9] Proceeding just as in the cases above, this implies that in equilibrium where Lender-U pools all borrowers, $C_P = 0$.

Note that in the same way, we can show that this result holds in an equilibrium where Lender-U pools just the h- and b-types, but not the l-types. ■

Candidate Equilibrium: Pool-1

Proposition 11 *A pure strategy equilibrium wherein Lender-U pools all borrowers occurs if* $\rho^I \geq \tilde{\rho}_P^1 (\equiv (\frac{1-\theta}{1-\mathbb{E}(\theta)})\rho^U)$ *and is characterized as follows:*
(a) Lender-U offers menu $(\underline{R}_l^I, 0)$.
(b) Lender-I offers $(\underline{R}_h^I, 0)$ *to h-types and* $(\underline{R}_l^I, 0)$ *to l-types.*
(c) All borrowers go to the Lender-U for loans.

[9] One can show this proceeding in a similar way as in Case 2 for Lemma 4. Also note that pooling is feasible only if $\nu_l \geq \nu_b + \nu_h$.

Proof. First, from lemma 10, it follows that the pooling offer is of the form $(R_P, 0)$. Since the pooling offer must yield non-negative profits, it must satisfy

$$[1 - \mathbb{E}(\theta)]R_P \geq \rho^U \tag{48}$$

where $\mathbb{E}(\theta) \equiv \nu_b\theta_b + \nu_h\theta_h + \nu_l\theta_l$ is the expected value of θ. This implies that pooling is feasible for contracts of the form $(R_P, 0)$ such that $R_P \geq \rho^U/[1 - \mathbb{E}(\theta)]$. Third, for a pooling contract $(R_P, 0)$ to hold, the entrant has to ensure that l-types accept its offer. Therefore, it must be true that $R_P \leq \underline{R}_l^I$, that is $\rho^I \geq (\frac{1-\theta_l}{1-\mathbb{E}(\theta)})\rho^U \equiv \tilde{\rho}_P^1$. Since increasing R_P increases profits, Lender-U offers $(\underline{R}_l^I, 0)$ such that the participation constraint of l-types just bind. ■

Candidate Equilibrium: Pool-2

Proposition 12 *A pure strategy equilibrium wherein Lender-U pools h-types and l-types occurs if* $\rho^I \geq \tilde{\rho}_P^2$ *and is characterized as follows:*
(a) Lender-U offers menu $(\underline{R}_h^I, 0)$.
(b) Lender-I offers $(\underline{R}_h^I, 0)$ *to h-types and* $(R_l^I, 0)$ *to l-types where* $\pi_l^U(R_l^0, C_l^0) = 0$, $V_l(R_l^0, C_l^0) = V_l(R_l^I, 0)$ *and* $V_l(\underline{R}_h^I, 0) = V_l(R_l^0, C_l^0)$.
(c) Both h-type from the b-type borrowers accept Lender-U's offer, while l-types accept Lender-I's offer.

Proof. Following the same procedure as above, we know that the pooling offer must yield non-negative profits. So it must satisfy

$$[1 - (\nu_b\theta_b + \nu_h\theta_h)]R_P \geq \rho^U \tag{49}$$

This implies that pooling is feasible for contracts of the form $(R_P, 0)$ such that $R_P \geq \rho^U/[1 - (\nu_b\theta_b + \nu_h\theta_h)]$. For this pooling contract $(R_P, 0)$ to hold, the entrant has to ensure that h-types accept its offer. Therefore, it must be true that $R_P \leq \underline{R}_h^I$, that is $\rho^I \geq (\frac{1-\theta_h}{1-(\nu_b\theta_b+\nu_h\theta_h)})\rho^U \equiv \tilde{\rho}_P^2$. Since increasing R_P increases profits, Lender-U offers $(\underline{R}_h^I, 0)$ such that the participation constraint of the h-types just bind. Likewise, Lender-I's offers $(R_l^I, 0)$ where $R_l^I \geq \bar{R}_l^I$ to l-types is exactly as given in Screen-2, and yields the best payoff that Lender-U could offer them. ■

Hybrid Equilibria

Hybrid equilibria has elements of pooling and screening. This occurs when Lender-U pools or bunches adjacent types but screens the non-adjacent types. The hybrid equilibria can be described as follows.

Candidate Equilibrium: Hybrid-1

Proposition 13 *A pure strategy equilibrium wherein Lender-U separates only the l-types and bunches (pools) the b-types and the h-types occurs when $\tilde{\rho}_S^{b,h} > \rho^I \geq \tilde{\rho}_S^{h,l}$ and $\rho^I \geq \tilde{\rho}_P^2$, and is characterized as follows:*
(a) Lender-U offers menu $\{(\underline{R}_h^I, 0); (R_l^U, C_l^U)\}$ given by (41).
(b) Lender-I offers $(\underline{R}_h^I, 0)$ to h-types and $(\underline{R}_l^I, 0)$ to l-types.
(c) The l-type accepts the offer (R_l^U, C_l^U) from Lender-U. The b-type and the h-type accept Lender-U's offer $(\underline{R}_h^I, 0)$.

Proof. Since $\tilde{\rho}_S^{b,h} > \rho^I \geq \tilde{\rho}_S^{h,l}$, Lender-$U$ cannot sort the h-types from b-types but can sort l-types from h-types. Lender-U offers (R_l^U, C_l^U) as given in (41). Just as in Screen-1, this is accepted by l-types and rejected by b-types and the h-types. Also, as was the case for Pool-2, for a sufficiently low ν_b and $\rho^I \geq \tilde{\rho}_P^2$, Lender-$U$ offers $(\underline{R}_h^I, 0)$, which is accepted by the b-type and the h-type and yields non-negative profits.[10] ∎

Candidate Equilibrium: Hybrid-2

Proposition 14 *A pure strategy equilibrium wherein Lender-U screens out the b-types and bunches the l-type and the h-type occurs when $\rho^I \geq \tilde{\rho}_Y$ is characterized as follows:*
(a) Lender-U offers menu (R_g^U, C_g^U) given by (42) and (43)
(b) Lender-I offers $(\underline{R}_h^I, 0)$ to h-types and $(\underline{R}_l^I, 0)$ to l-types.
(c) The h-type and the l-type accept the offer (R_g^U, C_g^U). The b-types reject this offer.

Proof. First, as l-types accept Lender-U's offer Lemma 3 must hold. Also, because Lender-U screens out the b-types, Lemma 4 must hold. Therefore, $V_l(R, C) = \bar{V}_l^I$ and $V^0 = V_b(R, C)$. Solving these two equations for (R, C), we get Lender-U's offers to be (R_g^U, C_g^U). Note that $V_h(R_g^U, C_g^U) > \bar{V}_h^I$, and therefore, h-types borrow from Lender-U. ∎

Appendix B: Model Solution

The set of pure strategy (candidate) equilibria for the model are given in Table 1. The next step is to determine which of these candidates emerge as the (final) equilibrium of the model for a given set of parameter values. The closed-form solutions to the lenders' offers in each equilibria are given by (41)-(45). These solutions are then used to derive Lender-U's profits from its offers in each candidate equilibria as a function of parameters. From the set of profit

[10] Note that, since $\rho^I > \rho^U$, Lender-U can make offers with $R_h^I < \underline{R}_h^I$, but $(\underline{R}_h^I, 0)$ maximizes the Lender-U's profits from this bunching.

functions, one for each of the equilibrium offers, Lender-U selects one that yields the maximum profits. This gives us the optimal lending behavior and the optimal contract offers for each lender for a given set of parameter values. Since the maximum is obtained over a finite set of values, each a function of the parameters, it provides us with a solution to the model.

I define parameters x, V^0 and the three different values of θ, namely θ_b, θ_h, θ_l to be the primitives of the model. The two boundary conditions are used to determine the maximum and minimum values for the lenders' cost of funds in terms of the primitives of the model as $\rho_{\min} \equiv (1-\theta_b)x - V^0$ and $\rho_{\max} \equiv (1-\theta_h)[x - \frac{V^0}{(1-\theta_b)}]$. In what follows, we fix the value of Lender-U's cost of funds and vary Lender-I's cost of funds. Therefore, while ρ^U is fixed, $\rho^I \in [\rho^I_{\min}, \rho^I_{\max}]$. Note that for a given set of primitives, the screening cutoffs $\tilde{\rho}_S^{h,l}$ and $\tilde{\rho}_S^{b,h}$ vary with β, and are given as before

$$\tilde{\rho}_S^{h,l} = \frac{1}{1-(1-\beta)\theta_l}\rho^U \tag{50}$$

$$\tilde{\rho}_S^{b,h} = \frac{\theta_b-\theta_h}{\theta_b(1-\theta_h)-\beta\theta_h(1-\theta_b)}\rho^U + \frac{(1-\beta)\theta_h(1-\theta_h)}{\theta_b(1-\theta_h)-\beta\theta_h(1-\theta_b)}[(1-\theta_b)x - V^0]. \tag{51}$$

Also, for given set of primitives, we define $\hat{\rho}^U$, such that[11]

$$\rho^U \gtreqless \hat{\rho}^U \Leftrightarrow \tilde{\rho}_S^{h,l} \gtreqless \tilde{\rho}_S^{b,h}$$

Therefore, for the same set of primitives, the nature of equilibria varies with different values of ρ^U.

Since there can be infinite variations in the set of parameter values, the exposition here is selective. The aim is to illustrate how institutional features and market conditions affect lending behavior of informed and uninformed lenders. In what follows, I first show how the equilibrium changes with different values of ρ^U that lie in the interval $[\rho^I_{\min}, \rho^I_{\max}]$. Thereafter, I discuss how this equilibrium changes with the salvage rate (value) of collateral, β, given that $\beta \in (0,1)$. Finally, changes in equilibrium are shown for different values of ν_b.

Graphical representation

I describe the solution to the model by demarkating the regions of the parameter space wherein a particular candidate emerges as the (final) equilibrium of the model. For simplicity of exposition, I present the equilibrium regions in (ν_h, ρ^I) space. Each figure comprises of four subplots obtained by varying parameters of the model as given above. Parameter values of $x = 16$,

[11] The closed form solution is $\hat{\rho}^U \equiv \frac{\frac{(1-\beta)\theta_h(1-\theta_h)}{\theta_b(1-\theta_h)-\beta\theta_h(1-\theta_b)}[(1-\theta_b)x-V^0]}{\left(\frac{1}{1-(1-\beta)\theta_l} - \frac{\theta_b-\theta_h}{\theta_b(1-\theta_h)-\beta\theta_h(1-\theta_b)}\right)}$. Strictly speaking, this cutoff $\hat{\rho}^U$ is an artifact of keeping ρ^U constant while varying ρ^I. Intuitively, this implies that the nature of equilibria are different for absolute values of the lenders' cost of funds.

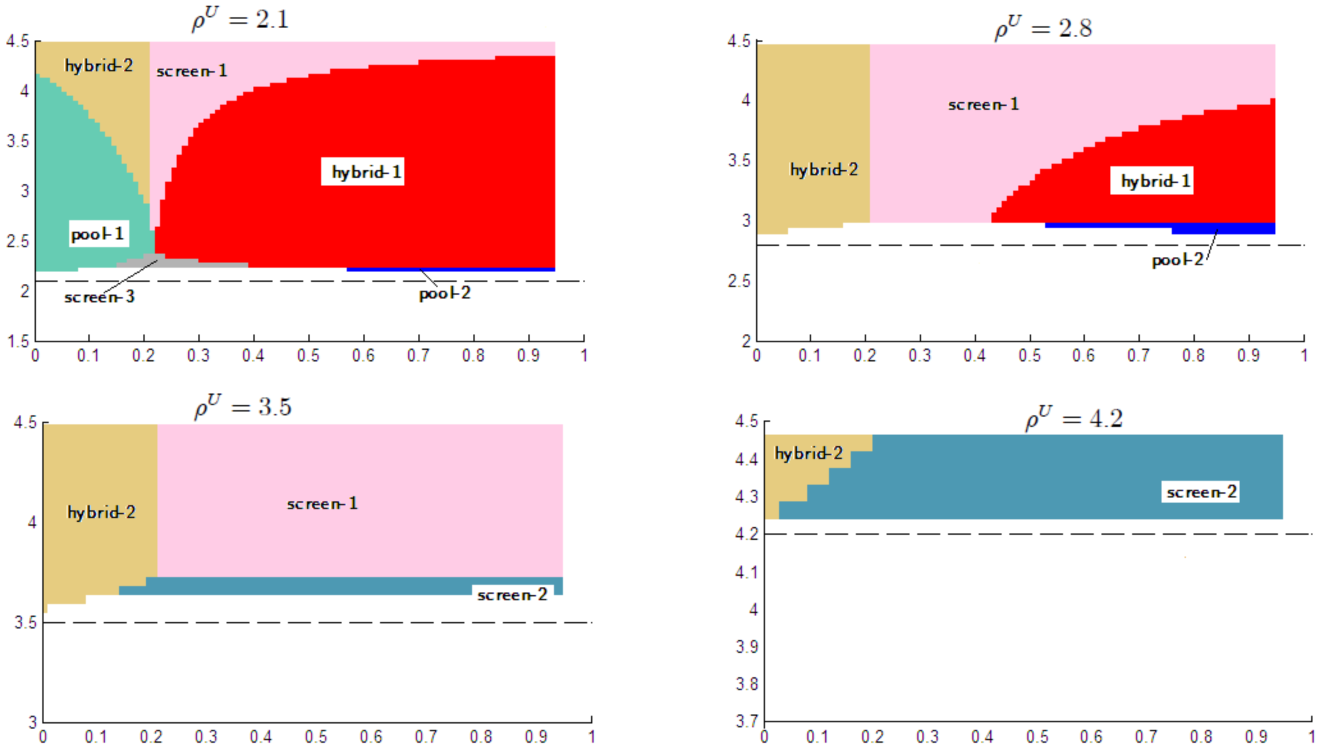

Figure 1: Solution in (ν_b, ρ^I) space for different values of ρ^U and low $\beta(= 0.25)$. The graphs are generated with $x = 16$, $V^0 = 3$, $\theta_b = 0.7$, $\theta_h = 0.25$, and $\theta_l = 0.08$. Also, $\beta = 0.25$ and $\nu_b = 0.05$. For this set of parameters, $\rho^I \in [1.8, 4.5]$ and $\hat{\rho}^U = 2.85$.

$V^0 = 3$, $\theta_b = 0.7$, $\theta_h = 0.25$, and $\theta_l = 0.08$, are used to generate the graphs in this section. The dotted lines in the graphs denote the uninformed lender's cost of funds, ρ^U. The colored regions denote equilibria in which the uninformed lender is able to secure at least one creditworthy borrower type. On the other hand, if the uninformed lender's cost advantage is sufficiently small, the informed lender dominates (i.e., all borrowers go to the informed lender for loans), as shown by the white region above (and below) the dotted line.

For $\nu_b = 0.05$, the Figures 1 and 2 show how the equilibria changes with ρ^U for high $\beta(= 0.75)$ and low $\beta(= 0.25)$ respectively. Evidently switching from a low value of β (Figure 1) to a high value of β (Figure 2) increases the likelihood of screening equilibria. However, even for a given value of β the equilibrium changes depending on the absolute value of ρ^U and the magnitude of the cost advantage $(\rho^I - \rho^U)$, and can be illustrated as follows.

For $\rho^U < \hat{\rho}^U$, it follows that $\tilde{\rho}_S^{b,h} > \tilde{\rho}_S^{h,l}$. With a large cost advantage such that $\rho^I > \tilde{\rho}_S^{b,h} > \tilde{\rho}_S^{h,l} > \rho^U$, Lender-$U$ can screen all borrowers; and for higher values of ν_h, sorting all borrower types under Screen-1 is optimal for Lender-U. However, for low values of ν_h, Lender-U screens out uncreditworthy types but pools creditworthy types under Hybrid-2. Equilibrium lending behavior changes with a smaller cost advantage such as when $\tilde{\rho}_S^{b,h} > \rho^I > \tilde{\rho}_S^{h,l} > \rho^U$. In this

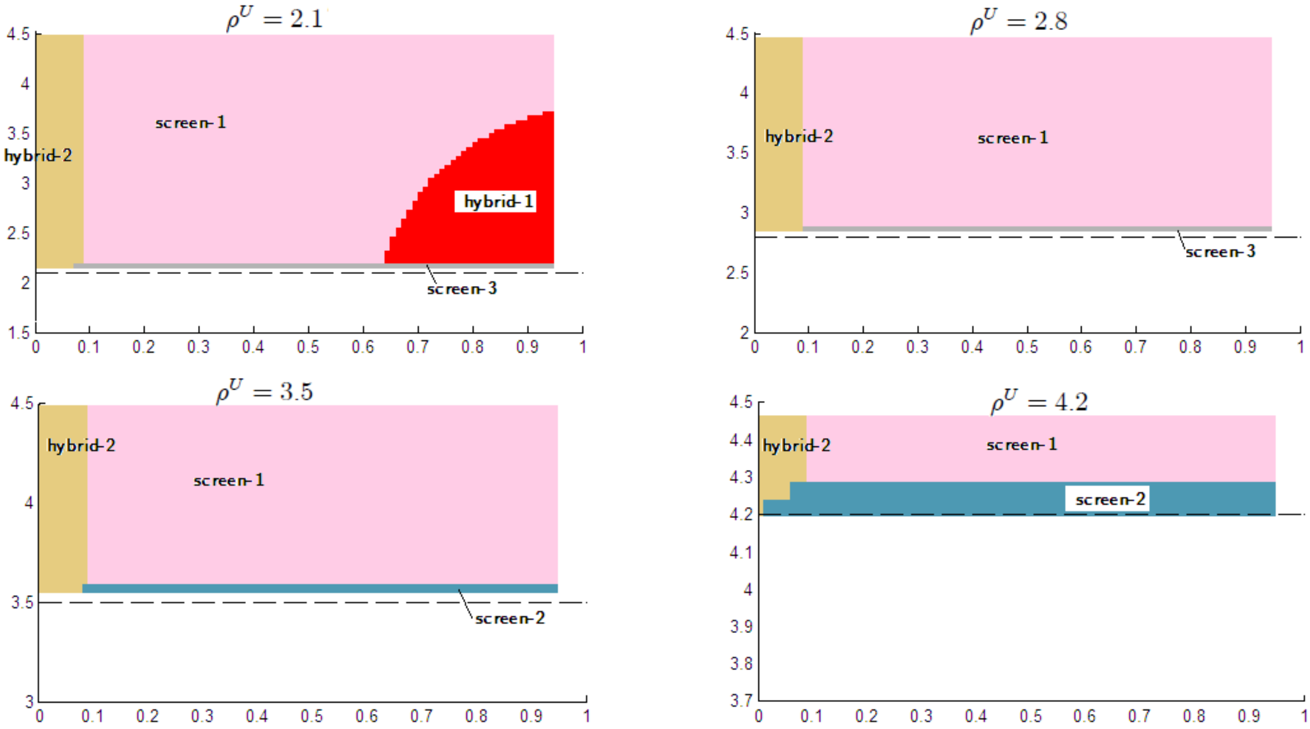

Figure 2: Solution in (ν_b, ρ^I) space for different values of $\rho^U$and high $\beta(= 0.75)$. The graphs are generated with $x = 16$, $V^0 = 3$, $\theta_b = 0.7$, $\theta_h = 0.25$, and $\theta_l = 0.08$. Also, $\beta = 0.75$ and $\nu_b = 0.05$. For this set of parameters, $\rho^I \in$[1.8, 4.5] and $\hat{\rho}^U = 2.9$.

case, Lender-U's cost advantage is just enough so that it can sort among creditworthy types, but not sufficiently large, so that it can screen out uncreditworthy types. Depending on the value of ν_h, three types of equilibria emerge: First, for low ν_h (and low β), Lender-U can find it optimal to pool all borrowers under Pool-1. Second, for intermediate values of ν_h, the equilibrium can be characterized by Screen-3, where the lenders split the market with l-types accepting the offer from Lender-U. Third, for high ν_h, Lender-U pools bad-risks with high-risk types under Hybrid-1 or Pool-2. Importantly, the region under Hybrid-1 or Pool-2 shrinks with increases in ρ^U and is replaced by Screen-1 and Screen-3 respectively.

For $\rho^U > \hat{\rho}^U$, it follows that $\tilde{\rho}_S^{h,l} > \tilde{\rho}_S^{b,h}$. Note that for large differences in the cost of funds, the equilibria are the same as in previous case. For smaller cost advantage, that is when $\tilde{\rho}_S^{h,l} > \rho^I > \tilde{\rho}_S^{b,h} > \rho^U$, Lender-$U$'s cost advantage is just large enough to screen out uncreditworthy types but not sort among good risks. This implies that Lender-U can secure the high-risk borrower and screen out the bad risks under Screen-2. This occurs typically for higher ν_h, because a larger proportion of h-types cannot be pooled with l-types.

Next, Figures 3 through 5 show how the equilibria change with β for a different values of ρ^U. For each of the Figures 3 through 5, we consider four values of β, namely 0.05, 0.1, 0.15

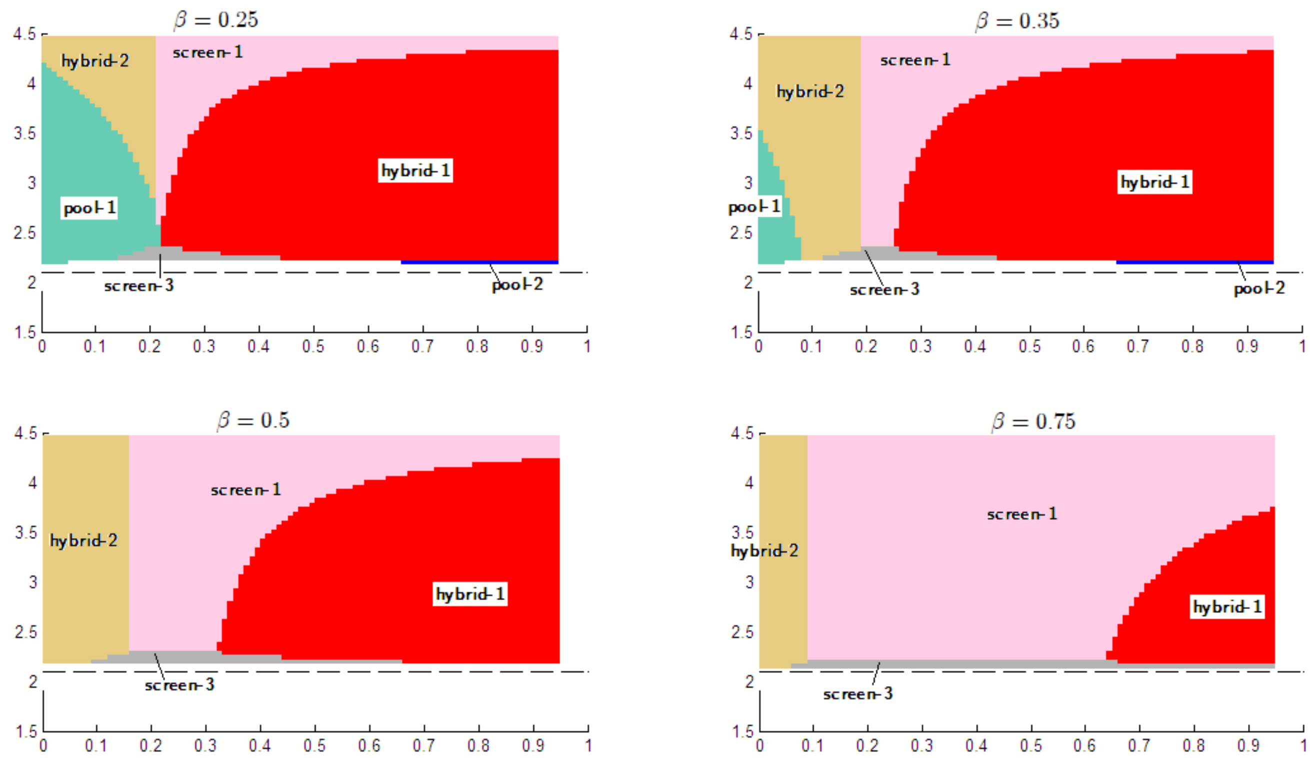

Figure 3: Solution in (ν_b, ρ^I) space for different values of β and low $\rho^U (= 2.1)$. The graphs are generated with $x = 16$, $V^0 = 3$, $\theta_b = 0.7$, $\theta_h = 0.25$, and $\theta_l = 0.08$. Also $\rho^U = 2.1$ and $\nu_b = 0.05$. For this set of parameters, $\rho^I \in [1.8, 4.5]$.

and 0.25, one for each of the subplots. Evidently, lower β implies a higher cost of screening and pooling equilibria dominate for these lower values. The general pattern of equilibria can be explained as follows. Higher β implies a greater salvage value of the collateral if the borrower defaults, and this increases the propensity for the screening behavior. However, for low ρ^U, such as $\rho^U = 2.1$ in Figure 3, Lender-U chooses pooling under favorable conditions (i.e., high ν_h) and this holds even for very high values of β. Likewise, for higher ρ^U, screening equilibria dominate and this holds even for very low values of β, as shown in Figures **??** ($\rho^U = 3.1$) and 5 ($\rho^U = 4.1$). Therefore, the cost of funds seem to be a critical determinant for the uninformed lender's choice between pooling and screening contracts. Low cost of funds induce pooling and can make it optimal for the lender to pool uncreditworthy types with creditworthy ones as shown by regions under Hybrid-1, Pool-1 and Pool-2 in Figures 3 through 5.

Finally, we will show how the equilibira change with ν_b for given β and ρ^U. We do this for four values of $\nu_b = 0.05, 0.1, 0.15$ and 0.25. For each of these values, we find the equilibria for three different values of $\beta = 0.25, 0.5$ and 0.75 and two values of $\rho^U = 2.3$ and 3.6. Not surprisingly, pooling uncreditworthy types is chosen for low ν_b whereas Lender-U finds it optimal to screen out bad-risks for higher ν_b. The principal difference between the equilibria at higher

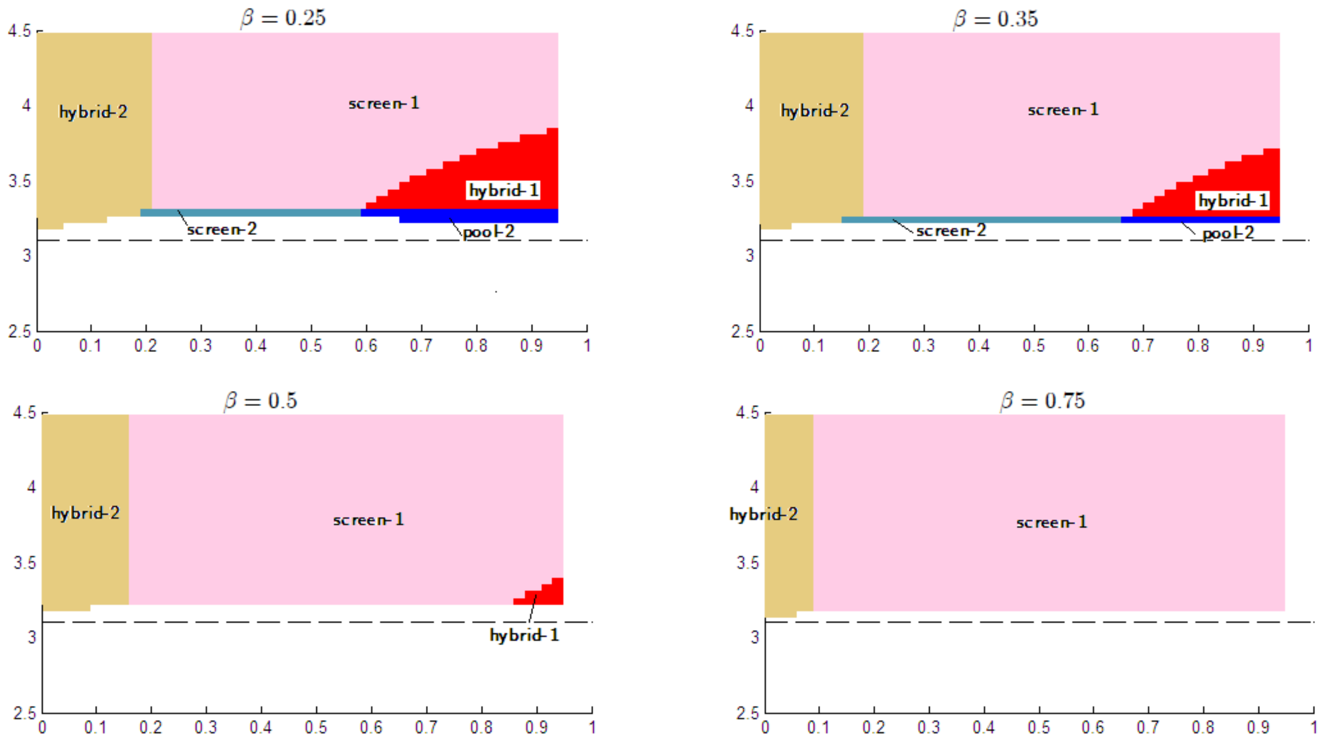

Figure 4: Solution in (ν_b, ρ^I) space for different values of β and moderate $\rho^U(= 3.1)$. The graphs are generated with $x = 16$, $V^0 = 3$, $\theta_b = 0.7$, $\theta_h = 0.25$, and $\theta_l = 0.08$. Also $\rho^U = 2.1$ and $\nu_b = 0.05$. For this set of parameters, $\rho^I \in[1.8,\ 4.5]$.

ρ^U and lower ρ^U is best seen for the case where Lender-U has a small cost advantage over Lender-I. For a higher $\rho^U(> \hat{\rho}^U)$, Lender-U can secure h-types but not l-types under Screen-2 whereas for a lower $\rho^U(< \hat{\rho}^U)$, Lender-U secures l-types but not h-types under Screen-3.

We do not show all of the equilbria calculated here (primarily because the results are qualitatively similar). With $\rho^U = 2.1$, the change in equilibria for different values of ν_b are given for $\beta = 0.25$ and $\beta = 0.75$ in figures 6 and 7 respectively. In figure 8, the changes in the equilibria are shown for $\beta = 0.5$ and $\rho^U = 3.6$. The results for $\beta = 0.25$ and $\beta = 0.75$ (not shown here) are similar.

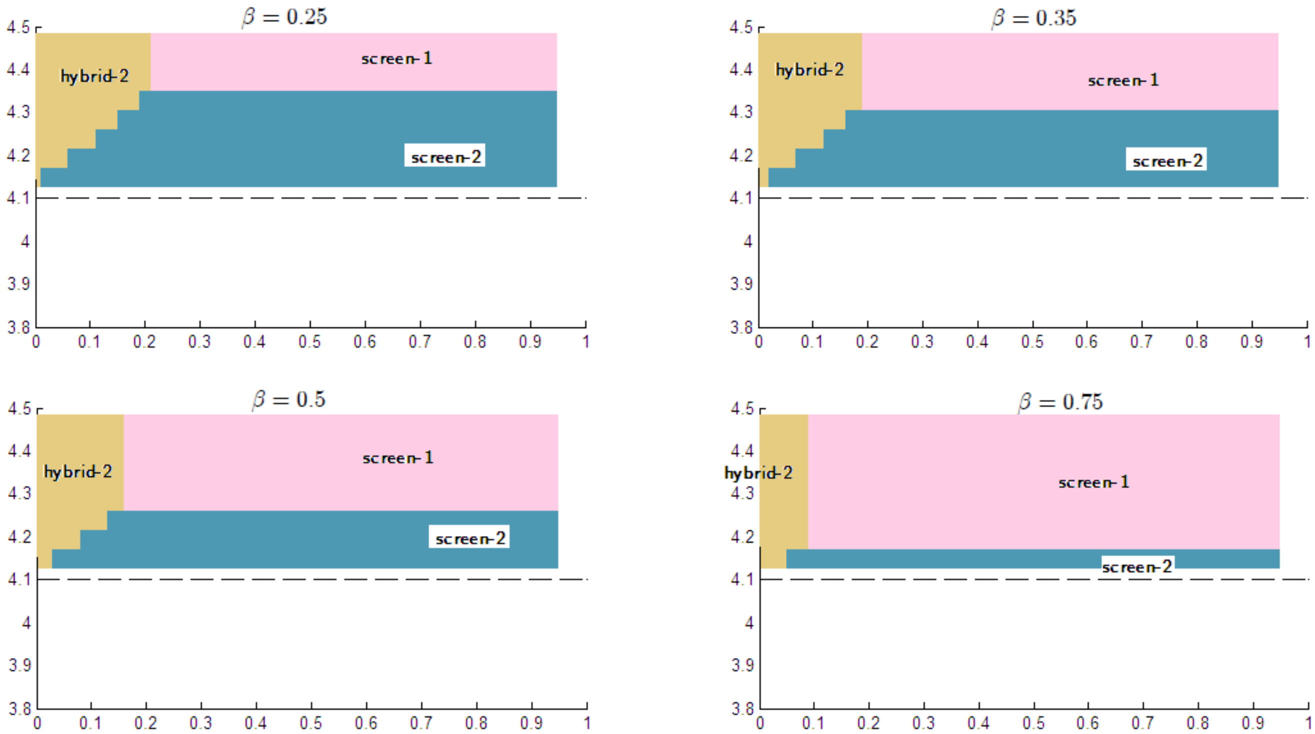

Figure 5: Solution in (ν_b, ρ^I) space for different values of β and high $\rho^U(= 4.1)$. The graphs are generated with $x = 16$, $V^0 = 3$, $\theta_b = 0.7$, $\theta_h = 0.25$, and $\theta_l = 0.08$. Also $\rho^U = 2.1$ and $\nu_b = 0.05$. For this set of parameters, $\rho^I \in [1.8, 4.5]$.

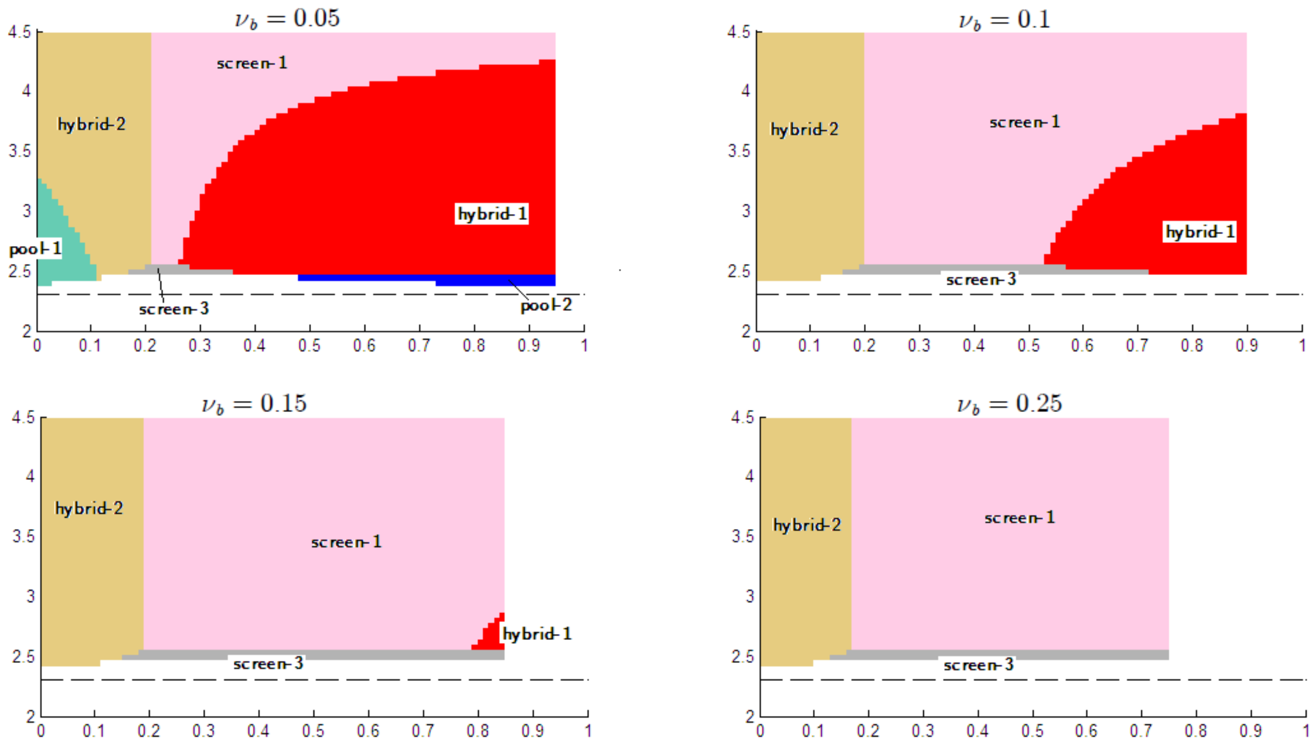

Figure 6: Solution in (ν_b, ρ^I) space for different values of ν_b and low $\rho^U (= 2.1)$ and low β. The graphs are generated with $x = 16$, $V^0 = 3$, $\theta_b = 0.7$, $\theta_h = 0.25$, and $\theta_l = 0.08$. Also $\rho^U = 2.1$ and $\beta = 0.25$. For this set of parameters, $\rho^I \in [1.8,\, 4.5]$ and $\hat{\rho}^U = 2.85$.

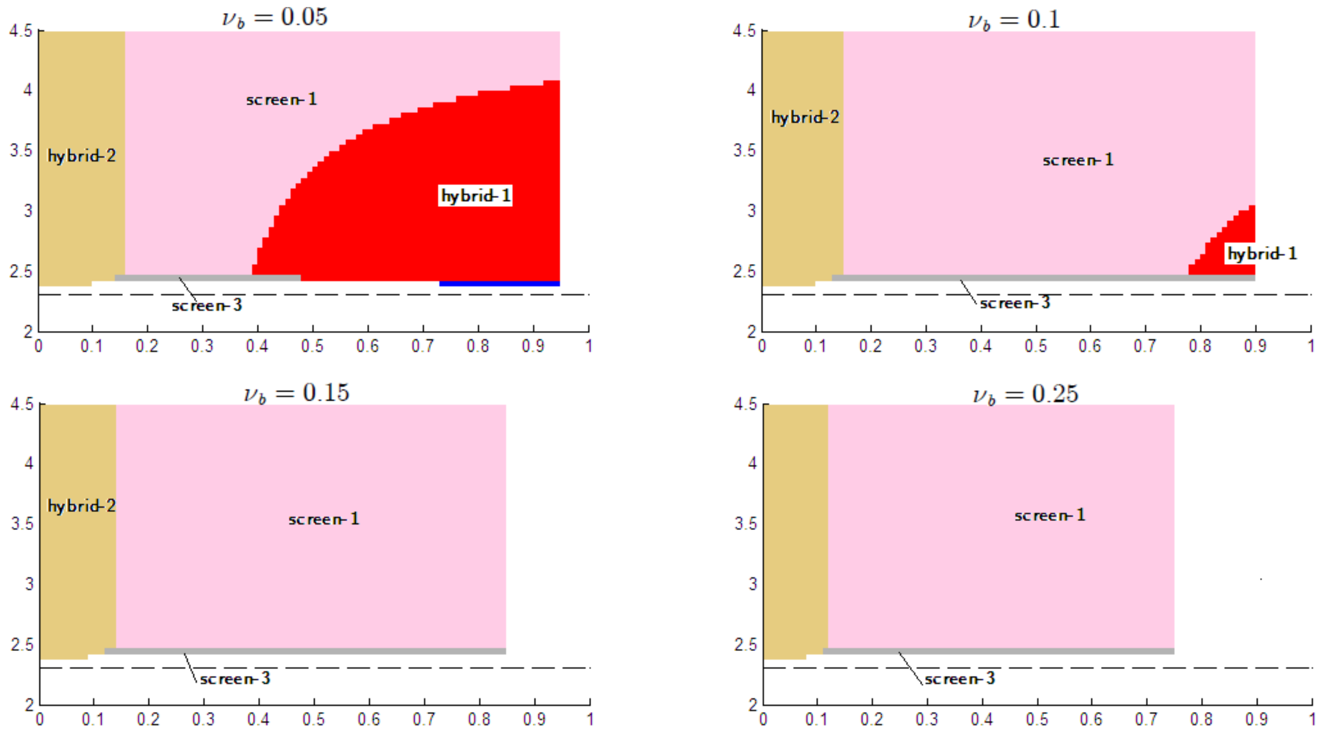

Figure 7: Solution in (ν_b, ρ^I) space for different values of ν_b and low $\rho^U(= 2.1)$ and moderate β. The graphs are generated with $x = 16$, $V^0 = 3$, $\theta_b = 0.7$, $\theta_h = 0.25$, and $\theta_l = 0.08$. Also $\rho^U = 2.1$ and $\beta = 0.5$. For this set of parameters, $\rho^I \in$[1.8, 4.5] and $\hat{\rho}^U = 2.88$.

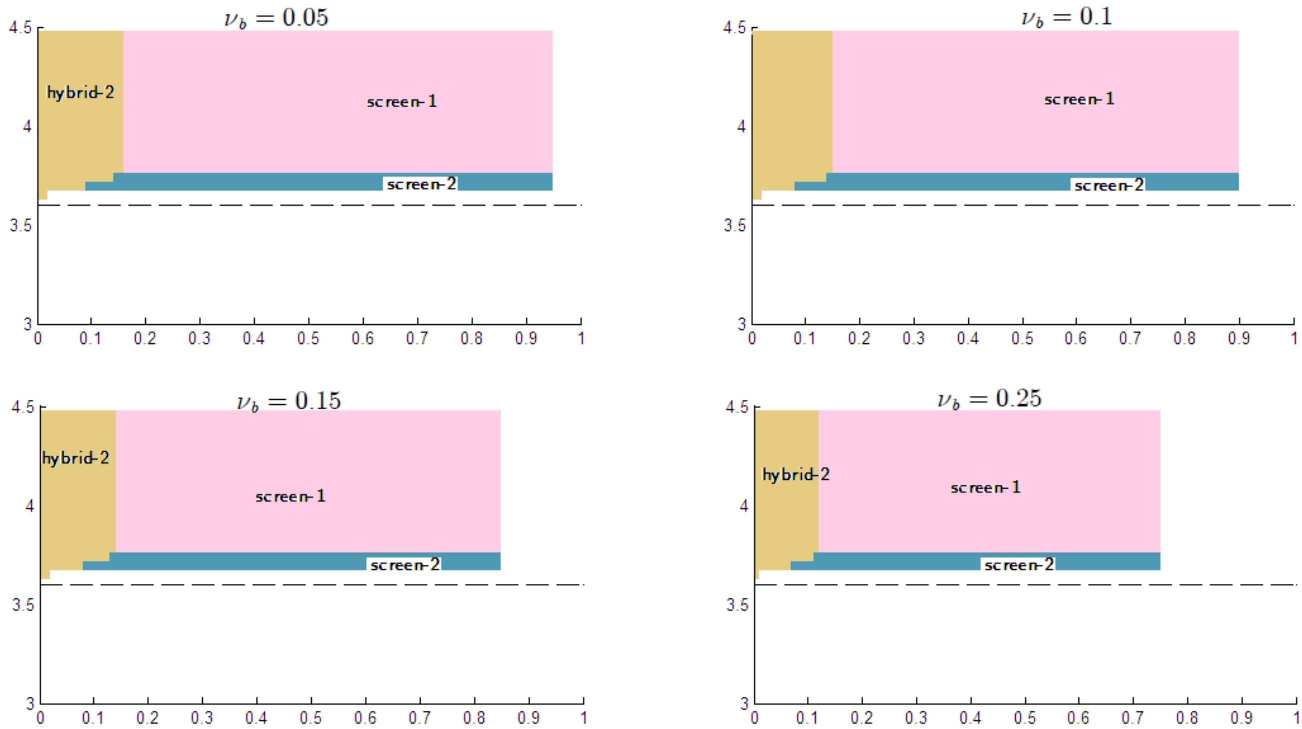

Figure 8: Solution in (ν_b, ρ^I) space for different values of ν_b and high $\rho^U(= 3.6)$ and moderate β. The graphs are generated with $x = 16$, $V^0 = 3$, $\theta_b = 0.7$, $\theta_h = 0.25$, and $\theta_l = 0.08$. Also $\rho^U = 2.1$ and $\beta = 0.5$. For this set of parameters, $\rho^I \in$[1.8, 4.5] and $\hat{\rho}^U = 2.9$.