Learning and Global Dynamics

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- The main idea is to study stability under learning of systems analyzed by Benhabib, Schmitt-Grohe, and Uribe (2001, JET and elsewhere).
- The learning dynamics give a different perspective from the RE dynamics.

What Benhabib, Schmitt-Grohe, and Uribe said

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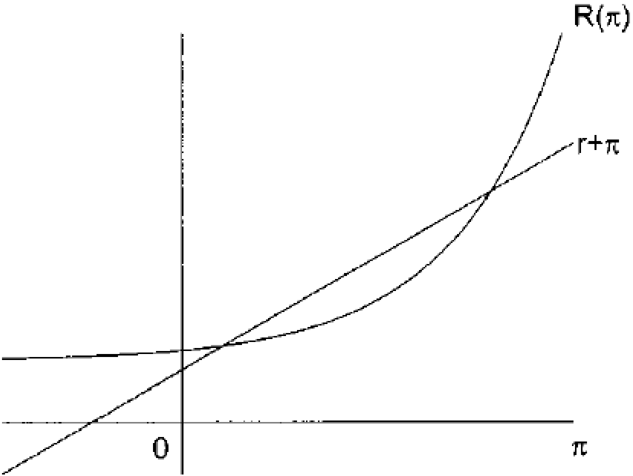
- Many analyses include a continuous, "active" Taylor type monetary policy rule $R'(\pi^\star) > 1$, where $\pi^\star$ is the target inflation rate of the monetary authority.
- Main point: This combination of assumptions always implies the existence of a second steady state inflation rate $\pi^L < \pi^\star$.
- Perfect foresight equilibria may exist in which inflation begins in the neighborhood of $\pi^\star$ but converges asymptotically to π^L along an oscillatory path.

Benhabib, et al.: Existence of a “liquidity trap”



Benhabib, et al., Figure 3

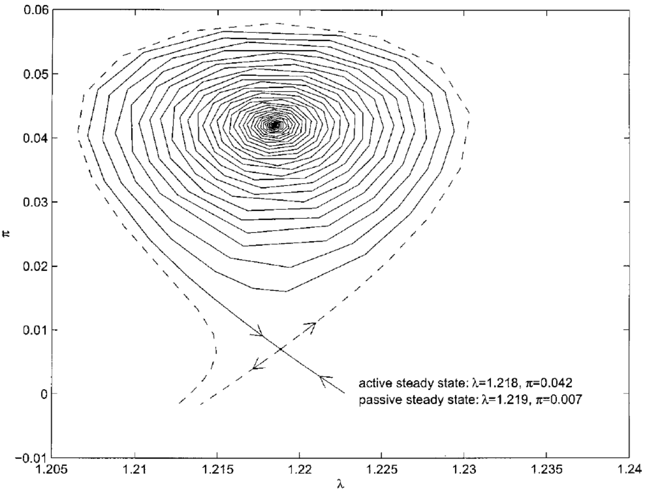
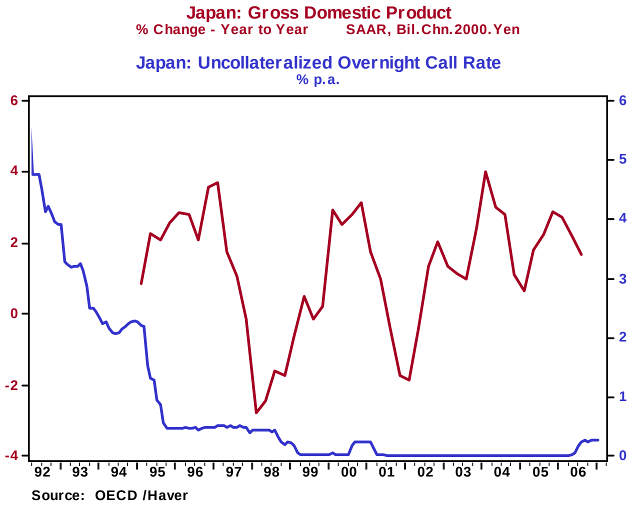


FIG. 3. Separable preferences: Saddle connection from the active to the passive steady state.

Japan


Japan: Gross Domestic Product
% Change - Year to Year SAAR, Bil.Chn.2000.Yen
Japan: Uncollateralized Overnight Call Rate
% p.a.
6
4
2
0
-2
-4
6
5
4
3
2
1
0
92 93 94 95 96 97 98 99 00 01 02 03 04 05 06
Source: OECD /Haver

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- How do policy choices influence these dynamics?

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- Under "normal policy," economy will converge to targeted steady state, and agents will behave as if they have rational expectations.
- Large, pessimistic shocks can send the economy on a path toward the low inflation steady state.
- Alternative policies may eliminate this possibility.

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- The labor market is competitive.

More background

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- Elasticity of substitution between goods is given by $\nu > 1$.
- Price adjustment costs are of the Rotemberg type.

- Households maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U_{t,j} \left(c_{t,j}, \frac{M_{t-1,j}}{P_t}, h_{t,j}, \frac{P_{t,j}}{P_{t-1,j}} - 1 \right) \tag{3}$$

subject to

$$c_{t,j} + m_{t,j} + b_{t,j} + \tau_{t,j} = m_{t-1,j}\pi_t^{-1} + R_{t-1}\pi_t^{-1} b_{t-1,j} + \frac{P_{t,j}}{P_t} y_{t,j}. \tag{4}$$

where

$$U_{t,j} = \frac{c_{t,j}^{1-\sigma_1}}{1-\sigma_1} + \frac{\chi}{1-\sigma_2}\left(\frac{M_{t-1,j}}{P_t}\right)^{1-\sigma_2} - \frac{h_{t,j}^{1+\epsilon}}{1+\epsilon} - \frac{\gamma}{2}\left(\frac{P_{t,j}}{P_{t-1,j}} - 1\right)^2. \tag{5}$$

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- Notation standard; last term in utility is Rotemberg cost of price adjustment.

Fiscal policy

- The government budget constraint is

$$b_t + m_t + \tau_t = g_t + m_{t-1}\pi_{t-1} + R_{t-1}\pi_t^{-1}b_{t-1} \tag{6}$$

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- Assume fiscal policy follows

$$\tau_t = \kappa_0 + \kappa b_{t-1} + \psi_t + \eta_t \tag{8}$$

a linear tax rule as in Leeper (1991).

Monetary policy

- Monetary policy follows a global interest rate rule

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- For some purposes

$$f(\pi) = (R^\star - 1)\left(\frac{\pi}{\pi^\star}\right)^{AR^\star/(R^\star-1)} \tag{10}$$

where $f'(\pi^\star) = AR^\star$ is assumed larger than β^{-1}.

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- The baseline analysis is under normal policy.

Equilibrium

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- Combine these three with the government budget constraint, the fiscal policy rule, the monetary policy rule, and market clearing.

Benhabib et al. 2001.

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- At deflationary steady states, $c > 0$ and $h > 0$ are unique provided π is close to one and $g > 0$.
- Corresponding stochastic steady states exist when the support of the exogenous shocks is sufficiently small.

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- Equilibrium dynamics can be analyzed by considering the equations for c and π alone, provided debt dynamics are stationary.
- The system can be written as

$$\begin{bmatrix} c_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} B_{cc} & B_{c\pi} \\ B_{\pi c} & B_{\pi\pi} \end{bmatrix} \begin{bmatrix} c^e_{t+1} \\ \pi^e_{t+1} \end{bmatrix} + \begin{bmatrix} G_{cu} & G_{c\theta} \\ G_{\pi u} & G_{\pi\theta} \end{bmatrix} \begin{bmatrix} u_t \\ \theta_t \end{bmatrix} + \begin{bmatrix} \tilde{k}_c \\ \tilde{k}_\pi \end{bmatrix}.$$

Determinacy

- If both eigenvalues of B lie inside the unit circle, a unique nonexplosive solution exists of the form

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- The remaining condition for determinacy is that fiscal policy is "passive" according to Leeper (1991), which means that $\left|\beta^{-1} - \kappa\right| < 1$.

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- Then the steady state with inflation at target $\pi = \pi^{\star}$ is locally determinate.
- And, the steady state with inflation $\pi = \pi_L$ is locally indeterminate.

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$$\pi_{t+1}^{e} = \pi_{t}^{e} + \phi_{t}\left(\pi_{t-1} - \pi_{t}^{e}\right), \tag{12}$$

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- Here ϕ_t is the gain sequence.

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- Theorems: LSL. Simulations: Constant gain.

Expectational stability

- Approximate $\pi_{t+1}^{-1} c_{t+1}^{-\sigma_1}$ by $\left(\pi_{t+1}^{e} \left(c_{t+1}^{e}\right)^{\sigma_1}\right)^{-1}$. This changes the dynamic system slightly. The linearization is unchanged.

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- The system is now the two altered equations for c, π,

$$\pi_t = F_\pi\left(\pi_{t+1}^e, c_{t+1}^e, u_t, \theta_t\right), \quad (14)$$

$$c_t = F_c\left(\pi_{t+1}^e, c_{t+1}^e, u_t, \theta_t\right). \quad (15)$$

the monetary policy rule, and the updating equations for expectations.

More on expectational stability

- The REE is said to be expectationally stable if the differential equation in notional time τ

$$\begin{bmatrix} d\pi^e/d\tau \\ dc^e/d\tau \end{bmatrix} = \begin{bmatrix} T_\pi\left(\pi^e, c^e\right) \\ T_c\left(\pi^e, c^e\right) \end{bmatrix} - \begin{bmatrix} \pi^e \\ c^e \end{bmatrix} \tag{16}$$

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- The condition is then that both eigenvalues of $B - I$ have real parts less than zero.
- Proposition 2. For $\gamma > 0$ sufficiently small, the steady state at $\pi = \pi^\star$ is locally stable under learning and the steady state at π_L is locally unstable, taking the form of a saddle point.

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- One could also claim that liquidity traps that come out of this model are theoretical curiosities that need not worry actual policymakers.
- The authors take a different course, pointing out that certain regions of instability exist.
- They want to design policy to eliminate these regions of instability.
- They simulate the global dynamics with larger values of $\gamma > 0$.

Figure 1

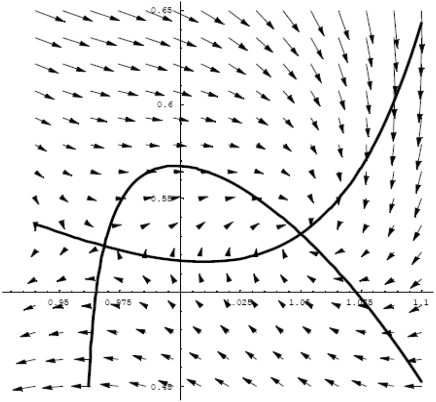

Figure 1: π^e and c^e dynamics under standard policy

More aggressive monetary policy

- Change the monetary policy rule to

$$R_t = \begin{cases} 1 + \theta_t f(\pi_t) & \text{if} \quad \pi_t > \tilde{\pi} \\ \hat{R} & \text{if} \quad \pi_t < \tilde{\pi} \end{cases}$$

and $\hat{R} \leq R_t \leq 1 + \theta_t f(\pi_t)$ if $\pi_t = \tilde{\pi}$.

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  and $\hat{R} \leq R_t \leq 1 + \theta_t f(\pi_t)$ if $\pi_t = \tilde{\pi}$.
- The authors choose $1 < \hat{R} < \min\left(1 + f(\pi_t), \beta^{-1}\tilde{\pi}\right)$.

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- Change the monetary policy rule to

$$R_t = \begin{cases} 1 + \theta_t f(\pi_t) & \text{if} \quad \pi_t > \tilde{\pi} \\ \hat{R} & \text{if} \quad \pi_t < \tilde{\pi} \end{cases}$$

  and $\hat{R} \leq R_t \leq 1 + \theta_t f(\pi_t)$ if $\pi_t = \tilde{\pi}$.
- The authors choose $1 < \hat{R} < \min\left(1 + f(\pi_t), \beta^{-1}\tilde{\pi}\right)$.
- The idea is to follow normal policy when $\pi_t \geq \tilde{\pi}$, but cut interest rates to a low level if inflation threatens to move below the threshold.

More aggressive monetary policy

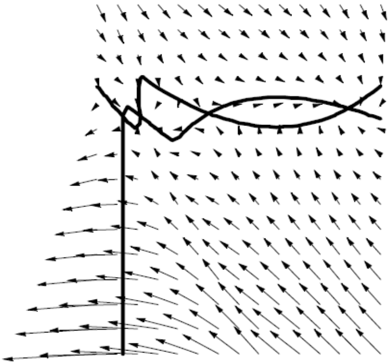

Figure 2: Four steady states with aggressive monetary policy but standard fiscal policy.

Altered monetary and fiscal policy

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- This can again create more than two steady states depending on the choice of $\tilde{\pi}$.

Figure 4

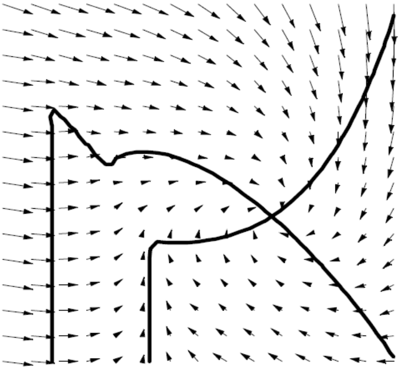

Figure 4: Unique steady state with a high value for $\tilde{\pi}$

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- The targeted, high inflation steady state would be locally stable in the learning dynamics.
- The possibility of deflationary spirals would still exist however, unless policy is chosen carefully.